1. Answer two simple questions below with brief explanations.

(a) (5%) Let us declare a multi-dimensional array by

```c
int x[m][n][p];
```

Write down the addressing formula for the element

\[ x[i][j][k] \]

if the starting address is \( a \).

(b) (5%) Given an expression of a mathematical formula as follows

\[ (A + B) \times (D + E) \div (F + A \times B) \]

write down its postfix expression.

2. Answer two questions about binary search tree.

(a) (5%) In a binary search tree, someone wants to search for an integer number on the tree and have an unfinished search sequence 2, 252, 401, 393, 330, 344, 307. Can you guess the range of the queried number? A brief explanation is necessary!

(b) (10%) Given a binary search tree, below is a code to find the minimum of the tree.

```c
TREE-MINIMUM(x)
while left[x] ≠ NIL
do x ← left[x]
return x
```

In the code, \( \text{left}[x] \) means the left child of node \( x \). Can you give a recursive version of the code, i.e., writing a recursive code to find the minimum of a tree?
3. (10%) Find the exact solution of the following recurrence equation,

\[ T(n) = 4T(n/2) + n^2, \quad T(1) = 1, \]

and prove your answer by mathematical induction. You only need to prove the cases when \( n = 2^k \).

4. (10%) Find the probability that a family with five children does not have a boy, if the sexes of children are independent and if the probability that the \( i \)th child is a boy is \( 0.5 \) and \( 0.5 \) otherwise.

5. (10%) Use mathematical induction to prove that \( 2^n < n! \) for every positive integer \( n \) with \( n \geq 4 \).

6. (a) (7%) How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points? Explain why?

(b) (8%) Show that among 100 people there are at least 9 who were born in the same month.

7. Link is an effective approach to construct cyclic graph directory structure from tree structure base. Explain the difference between symbolic link and non-symmetric link (hard link):

   (a) (4%) When they are deleted, will the linked files erased too?
   (b) (4%) Which one of them suffers dangling pointer problem? How a dangling pointer happens?

8. Answer the following questions concerning memory management.

   (a) (6%) Give your definitions to internal and external fragmentation.
   (b) (6%) How to solve the memory waste problem caused by external fragmentation if contiguous-memory allocation scheme is used?

9. Answer the following questions concerning process.

   (a) (4%) Briefly explain how a thread differs from a process.
   (b) (6%) List at least 3 events that might occur to cause context switch to a running user process.