1. (20%) Convert the following decimal numbers to hexadecimal and binary.
   (a) 229
   (b) 45816

2. (10%) Simplify the Boolean expression AB + A (B + C) + B (B + C).

3. (10%) For the Boolean function F = A'B'C' + B'CD' + A'BC'D' + AB'C'
   (a) Draw the K-map to find the simplified form.
   (b) Draw the two-level gate implementation of the simplified function.

4. (10%) Describe how to realize f(a, b, c) = \Sigma m(2, 4, 5, 7) with a 4-to-1 multiplexer module.

5. (10%) A sequential circuit with one D flip-flop and two inputs x and y is specified by the following input equations:
   \[ D_A = A \oplus x \oplus y \]
   The D_A symbol implies a D flip-flop with output A. The x and y variables are the inputs to the circuit.
   (a) Draw the logic diagram of the circuit.
   (b) Derive the state diagram of the circuit.

6. (20%) Consider a BCD synchronous counter by using T flip-flops.
   (a) Derive the state table.
   (b) Draw the logic diagram of the circuit.

7. (20%) It is necessary to formulate the Hamming code for four data bits, D_6, D_5, D_6, and D_7, together with three parity bits, P_1, P_2, and P_8.
   (a) Evaluate the 7-bit composite code word for the data word 1100.
   (b) Evaluate three check bits, C_6, C_5, and C_7, assuming no error.
   (c) Assume an error in bit D_6 during writing into memory. Show how the error in the bit is detected and corrected.
   (d) Add parity bit P_8 to include double-error detection in the code. Assume that error occurred in bit P_2 and D_6. Show how the double error is detected.