1. 總分 100 分。

Problem 1:
A random variable X has a probability density
\[ f(x) = \begin{cases} \alpha x & \text{for } 0 < x < 4 \\ 0 & \text{elsewhere} \end{cases} \]

(a) Find \( \alpha \). (3%)
(b) Find \( P(1 < x < 3) \). (3%)
(c) Find \( P(x=2) \). (2%)
(d) Find its expected value \( E(x) \). (3%)
(e) Find its variance \( V(x) \). (4%)

Problem 2: 請將選擇題答案書寫於答案卷內，於題目後答不予計分。
True or false
(a) \( A \cdot B \times C \) equals to \( C \cdot A \times B \). \( \Rightarrow \) (T) true, (F) false. (2%)
(b) \( (A \times B) \times C \) is a scalar. \( \Rightarrow \) (T) true, (F) false. (2%)
(c) \( A \times (B \times C) + B \times (C \times A) + C \times (A \times B) = 0 \). hint: \( A \times (B \times C) = (A \cdot C)B - (A \cdot B)C \) \( \Rightarrow \) (T) true, (F) false. (2%)
(d) If \( A \) is perpendicular to \( B \) and \( C \) as well, then \( B \) is perpendicular to \( C \). \( \Rightarrow \) (T) true, (F) false. (2%)

Problem 3:
For points \( A = 2ax + ay - az, B = 4ax + 2ay + az, \) and \( C = -ax + ay - 2az, \)
(a) Find the unit vectors of \( B-A \) and \( C-A \). (6%)
(b) Find the unit vector normal to the plane spanned by \( B-A \) and \( C-A \). (6%)

Problem 4:
Find the general solution of the equation: \( xy + y + 3 = 0 \). (15%)

Problem 5:
Given \( A = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{pmatrix} \)
(a) Compute \( A^{-1} \). (5%)
(b) Compute the determinant of \( 2A^T A^{-1} \). (5%)
(c) Let \( S \) be a subspace of \( R^3 \) spanned by the third column of \( A \), find \( S^\perp \). (5%)
(d) How many linearly independent eigenvectors does \( A \) have? (5%)

Problem 6:
Solve the differential problem \( x'' + 4x = f(t) \) where
(a) \( f(t) = 0, x(0) = 1, x'(0) = -4 \). (8%)
(b) \( f(t) = \sin 3t, x(0) = x'(0) = 0 \). (8%)

Problem 7:
Given the Fourier transform pair: \( x(t) \leftrightarrow X(\omega) \), derive the Fourier transform of \( x(t-\alpha) \) and \( \frac{dx(t)}{dt} \). (14%)

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