Statistics

1. (10%) A bag contains seven coins — five fair coins (fair coin — a head on one side and a tail on the other side), one coin with a head on both sides, and one with a tail on both sides. Randomly select one coin from the bag and flip the selected coin. Given that the flip results in a head, what is the probability that the selected coin is a fair coin.

2. (10%) A random sample \( \{X_1, \ldots, X_{100}\} \) of size 100 is drawn from a normal population with unknown mean \( \mu \) and known variance \( \sigma^2 = 200^2 \). Suppose sample mean \( \bar{X} = \frac{\sum_{i=1}^{100} X_i}{100} = 25 \) is observed. Can we accept the hypothesis that \( \mu = 40 \)? Support your answer.

3. (30%) A quality control engineer tries to perform a hypothesis testing on the defective rate \( p \) of a production line. The engineer keeps inspecting the items until he finds the first defective. Let \( T \) denote the total number of items the engineer inspects. The engineer wants to test the hypothesis \( H_0 : p \geq 10\% \) against \( H_1 : p < 10\% \).

   (a) Which of the following regions is an appropriate critical region? Why?

   i. \( \{T \geq k\} \) for some constant \( k \).
   ii. \( \{T < k\} \) for some constant \( k \).

   (b) What is the probability distribution of \( T \)?

   (c) Find the type I error rate of the test for \( k = 15 \).

4. (20%) State whether each of the following variables is quantitative or qualitative and indicate the measure scale being used.

   (a) annual sales
   (b) soft-drink size (small, medium, or large)
   (c) earnings per share
   (d) method of payment (cash, check, credit card)

5. (10%) A sample of ten stocks on the New York Stock Exchange shows the following price-earning ratios

   \[ 9 \ 4 \ 6 \ 7 \ 3 \ 11 \ 4 \ 6 \ 4 \ 7 \]

   Compute the mean, median, mode, range, variance, and standard deviation.

6. (20%) Suppose \( P(A) = .40, P(A\mid B) = .60 \) and \( P(B\mid A) = .30 \).

   (a) Find \( P(A \cap B) \) and \( P(B) \).

   (b) Are events \( A \) and \( B \) independent? Why or why not?