總分 100 分，請依序作答，並詳列計算過程。

1. (22%) A Bingham plastic fluid flows in a pipe and it is observed that the central plug is 30mm in diameter when the pressure drop is 100 Pa/m. Given that the plastic viscosity is 0.6 Pa·s and the radius of pipe is 60mm.
   (a) Calculate the yield shear stress.
   (b) Show the distribution of velocity in the pipe.

2. (14%) A thick-walled cylindrical tubing of stainless steel having an inside radius of 5 mm and an outside radius of 10 mm is being used as a temporary cooling coil in a bath. The thermal conductivity of stainless steel is 21.5 \( \text{W/m} \cdot \text{K} \). The tube is covered with a 10-mm-thick insulation layer with the thermal conductivity of 0.2 \( \text{W/m} \cdot \text{K} \). Saturated steam at 130°C is flowing rapidly inside. The outside surrounding air is at 27°C. The convective coefficient for the inside steam surface of the pipe is 5500 \( \text{W/m}^2 \cdot \text{K} \), and the convective coefficient on the outside of the insulation layer is 10 \( \text{W/m}^2 \cdot \text{K} \). Calculate the heat loss for 1 m of pipe.

3. (14%) By using dimensionless analysis, derive the equation for predicting mass-transfer coefficients under forced convection. (Hint: find the equation relating Sherwood number, Schmidt Number and Reynolds number).

4. (20%) Solve the initial value problem: \( x^2y'' - 2xy' + 2y = 10 \sin(\ln(x)) + 6x^3 \), \( y(1) = 4 \), \( y'(1) = 4 \).

5. (20%) Solve the boundary value problem using separation of variables:
   \[
   \frac{\partial^2 \theta}{\partial t^2} - a^2 \frac{\partial^2 \theta}{\partial x^2} = 0 \quad 0 < x < 2, \quad t > 0,
   \]
   \[
   \theta(0, t) = \theta(2, t) = 0 \quad t \geq 0,
   \]
   \[
   \theta(x, 0) = 0, \quad \frac{\partial \theta(x, 0)}{\partial t} = 1 \quad 0 \leq x \leq 2.
   \]

6. (10%) A wire is bent into the shape of the quarter circle \( C \) given by \( x = \cos(t), \ y = \sin(t), \ z = 4 \) for \( 0 \leq t \leq \pi/2 \). The density function is \( \rho(x, y, z) = x^2y \) (g/cm).
   Please find the mass of the wire.