Problem 1: (20 points) 【證明題】

The unilateral Laplace transform is defined as shown below

\[ L(f(t)) = F(s) = \int_0^\infty f(t)e^{-st}dt, \]

and define \( f(0^-) \) be the pre-initial value and \( f(0^+) \) the post-initial value of \( f(t) \) at the initial time \( t = 0 \).

(a) Prove that \( L(f'(t)) = sF(s) - f(0^-) \). (7 points)

(b) Prove that \( f(0^+) = \lim_{s \to \infty} sF(s) \). (7 points)

(c) Prove that \( L(\int_0^t f(t)dt) = \frac{F(s)}{s} \). (6 points)

Problem 2: (30 points and 3 points each)

Answer each question "Yes" or "No". Just write down your answer, there is no need to specify the reasons.【是非題：只需回答是(Yes)或否(No)，無需說明任何理由或計算過程，將是非題答案填寫於答案紙內，於試題紙上作答不予計分】

(1) A system with input \( u(t) \) and output \( y(t) = |u(t)| \) is a linear system.

(2) A system with input \( u(t) \) and output \( y(t) = u(t-T), T > 0 \) is a causal system.

(3) Given the Laplace transform \( L(f(t)) = F(s) = \frac{a}{s(a^2 + a)}, a > 0 \), then

\[ \lim_{s \to 0^+} f(t) = \lim_{s \to 0^+} sF(s) = 1. \]

(4) For a linear time-invariant (LTI) system, the system's step response is the inverse Laplace transform of the system transfer function.

(5) For a linear time-invariant system, if it is BIBO (bounded-input bounded-output) stable, then it is also asymptotically stable.

(6) Given a linear time-invariant stable type-1 unity negative-feedback system, the steady state error of a step response is zero.

For Problems (7–10), let’s consider an unity negative-feedback system with loop transfer function \( L(s) = \frac{K}{(s+2)(s+10)}, K > 0 \).

(7) If the gain \( K \) is increasing, then the gain margin \( GM \) is decreasing.

(8) If the gain \( K \) is increasing, then the phase margin \( PM \) is also increasing.

(9) If the gain \( K \) is increasing, then the gain crossover frequency \( \omega_c \) is also increasing.

(10) The closed-loop system is an over-damped system for all \( K > 0 \).
Problem 3: (13 points)

Consider the following periodic function $f(t)$ in one period described as

![Diagram of a periodic function]

Find its Laplace transform (i.e., $F(s)$).

Problem 4: (13 points)

Find the relation between $R(s)$ and $Y(s)$.

![Control system diagram]

Problem 5: (24 points)

Consider a negative unit feedback system with forward transfer function $G(s) = ke^{-0.2s}/[s(1 + 0.1s)]$. (a) For $k = 2.5$, find the GM (gain margin), PM (phase margin), $\omega_s$ (gain-crossover frequency) and $\omega_p$ (phase-crossover frequency) (16 points). (b) Determine the value of $k$ such that the closed-loop system is unstable (8 points).