1. An electromechanical system shown in Figure 1 represents a moveable-plate capacity. Assume that the plate \( a \) of the parallel capacitor is fixed and the plate \( b \) with mass \( M \) is moved by force \( f \). If the capacitor \( C(d) = \frac{\epsilon A}{d} \), where \( \epsilon \) is the dielectric constant, \( A \) is the surface of the plates, and \( d \) is the separation distance of the plates, then the electric field produces a force opposing the motion of the plates, and it is related to the charge \( q \) across the plates: \( f_e = \frac{q^2}{2\epsilon A} \).

(a) Find the differential equations of this system. \hspace{1cm} (10\%)
(b) Find the Laplace transforms of the differential equations in part (a). \hspace{1cm} (10\%)

![Figure 1](image_url)

2. A unity feedback control system shown in Figure 2(a) is designed so that its closed-loop poles lie within the region shown in Figure 2(b).

(a) Find the natural frequency \( \omega_n \) and the damping ratio \( \zeta \). \hspace{1cm} (5\%)
(b) If \( K_p = 2 \) and \( p = 2 \), then find the values for \( K \) and \( K_f \). \hspace{1cm} (5\%)
(c) Show that, regardless of values \( K_p \) and \( p \), the controller can be designed to place the poles anywhere in the left side of the s-plane. \hspace{1cm} (3\%)

![Figure 2(a)](image_url)
3. Figure 3 shows a block diagram, where $G_p(s) = \frac{1000K}{s(s + a)}$. Design $G_D(s)$ and $G_c(s)$ so that the following performance specifications are satisfied.

- Ramp-error constant $K_r = 100$ when $a = 100$
- Rise time $T_r < 0.3$ sec
- Maximum overshoot < 8%
- Dominant characteristic equation roots $= -5 \pm 5j$
- System must be robust when $a$ varies between 8 and 12 (15%)
4. The feedback $H = (1 + K_s)$ as shown below can represent a parallel combination of direct feedback and minor loop rate feedback. If $G(s) = K((s^2 + 2s + 3.25)$ represents a spring-mass-damper system with a position output and a force input:
(a) Find the constraints on $K$ and/or $K_s$ for a steady-state error of 10% following step inputs. (8%)
(b) Calculate the value of $K_s$ for a damping ratio 0.707 from the quadratic characteristic equation, and use the corresponding roots in sketching the loci for varying $K_s$. (7%)

5. For the system shown below with $G(s) = (s + 1)/(s + 2)$:
(a) Find the value of $K$ required for a system time constant $T = 0.667$ sec. (8%)
(b) Calculate the corresponding unit step response. (7%)

6. For the system shown below with $G(s) = 1/[s(s+1)(s+4)]$:
(a) Find the limit on $K$ for stability. At this limit, determine the position of the system poles on the imaginary axis of the s-plane. (10%)

(b) Determine the limit on the value of $K$ for stability if the amplifier $K$ of part (a) is replaced by a dynamic compensator

$$K(0.5s + 1)$$
$$0.1s + 1$$

Where on the imaginary axis is a root pair located at this limit?