1. Please derive $P_fV_f' = P_fV_f''$ for the adiabatic reversible process, where $P_i$, $V_i$, $P_f$, and $V_f$ represent the pressure and volume in the initial and final state, respectively. $\gamma = C_p/m / C_v/m$ for an ideal gas. (10 points)

2. An ideal gas undergoes a single-stage expansion against a constant external pressure $P_{\text{external}} = P_f$ at constant temperature from $T_i$, $P_i$, $V_i$ to $T$, $P_f$, $V_f$.

(a) What is the largest mass $m$ that can be lifted through the height $h$ in this expansion? (5 points)

(b) The system is restored to its initial state in a single-state compression. What is the smallest mass $m'$ that must fall through the height $h$ to restore the system to its initial state? (5 points)

(c) If $h = 15.5$ cm, $P_i = 1.75 \times 10^6$ Pa, $P_f = 1.25 \times 10^6$ Pa, $T = 280$ K, and $n = 2.25$ mol, calculate the values of the masses in part (a) and (b) (5 points)

3. 21.05 g of steam at 373 K is added to 415g of H₂O (l) at 298 K at a constant pressure of 1 bar. Is the final state of the system steam or liquid water? Calculate $\Delta S_{\text{total}}$ for the process. $C_{p,iq}(\text{H}_2\text{O}, \text{liquid}) = C_{p,\text{vap}}(\text{H}_2\text{O}, \text{vapor}) = 75.3 \text{ J/Kmol} ; \Delta H_{\text{vaporization}}(\text{H}_2\text{O}) = 40650 \text{ J/mol}$. (10 points) 

4. Assume that 1-bromobutane and 1-chlorobutane from an ideal solution. At 273K, $P^*_{\text{chloro}} = 3790$ Pa and $P^*_{\text{bromo}} = 1394$ Pa. When only a trace of liquid is present at 273 K, $y_{\text{chloro}} = 0.75$.

a. Calculate the total pressure above the solution. (5 points)

b. Calculate the mole fraction of 1-chlorobutane in the solution. (5 points)

c. What value would Zchloro (the total 1-chlorobutane ratio in the system) have in order for there to be 4.86 mol of liquid and 3.21 mol of gas at a total pressure equal to that in part (a)? (5 points)
5. (30 points) Ten Short Answer Questions with Essential Written Procedures: (Fill in blanks, with attached calculation or reasoning. Each one is counted towards 3 points)

(1) The distribution function (probability density) of molecular speeds in an ideal gas is F, where F= F(v) and v is molecular speed. Define root-mean-square speed, \( v_{rms} \), with F(v): \[ F(v) \]

(2) The root-mean-square speed of hydrogen gas molecules (molar mass=2 g/mol) at 298 K is 1920 m/s, then the root-mean-square speed of oxygen gas molecules is _____ m/s.

(3) Transition-state theory predicts the rates of chemical reactions, by considering the equilibrium process leading to the transition state and the time spent at the transition state. The time of transition state is related to the vibrational frequency of transition state, \( \nu \), at temperature T, in which \( \nu \) is equal to _____.

(4) If the reaction \( 2A \rightarrow \) products is second order with a rate constant= \( k_2 \), then the rate of A disappearance in terms of \([A]\): \[ -d[A]/dt = _____. \] (note:[A]= concentration of reactant A at time t)

(5) Continuing the above problem, if the initial concentration at t=0 is \([A]_0\), the mathematical manipulation yields the half-life of A equal to _____.

(6) The reaction \( A \rightarrow P \) occurs on the surface of solid catalyst (E) is neither first nor zeroth order, following the Michaelis-Menten mechanism shown below: A + E \( \rightleftharpoons \) AE \( \rightarrow \) P + E. The reaction rate \( d[P]/dt \) is represented as a function of \([A]\) and two experimental parameters a and b: _____.

(7) The normalized wave function for an electron in 2s orbital of an excited hydrogen atom is \( \psi (r) \), where \( r \) = radial distance from the nucleus. The probability of the electron lying between \( r=a_0 \) and \( r=5a_0 \) (where \( a_0 \) is Bohr radius) is equal to _________.

(8) In hydrogen atom, the transition energy for an electron in ground state going to the 2s excited state is _____ eV.

(9) In hydrogen atom, an electron in 2p\(_x\) orbital of an excited hydrogen atom has an angular momentum equal to _____.

(10) The carbon monoxide molecule has a moment of inertia, I. Its lowest non-zero rotational energy level, corresponding to rotational quantum number equal to 1, has a rotational energy = _____.
6. (10 points) For an ionic crystal the bonding energy (or lattice energy) per ion-pair is given by the potential energy, U, as a function of the interionic distance, r, of an ion-pair:

\[ U = -\alpha z^2 e^2 / (4\pi \varepsilon_0 r) + B/r^6 \]

where \( \alpha \) is a constant dependent on crystal structure, \( z \) the valence of anion or cation, \( e \) the electron charge, \( \varepsilon_0 \) the permittivity in vacuum, and \( B \) a constant larger than 0. \( (4\pi \varepsilon_0)^{-1} = 9\times10^9 \) Nm\(^2\)/C\(^2\). For sodium chloride (FCC crystal), \( \alpha = 1.763 \), \( z = 1 \).

(1) Draw schematically \( U \) versus \( r \).(3 points)

(2) If the equilibrium distance between cation and anion (\( r_0 \)) is 0.283 nm, find the value of \( B \).(4 points)

(3) The ionic radius of chloride (atomic number= 17) is 0.167 nm, and that of sodium (atomic number= 11) 0.116 nm. Is the crystal closely packed? Why or why not? Why is the ionic radius of Cl\(^-\) much greater than that of Na\(^+\) ?(3 points)

7. (10 points) Given the steady-state three-dimensional Schrödinger equation:

\[ -\hbar^2/(8\pi^2 m) \nabla^2 \Psi + U \Psi = E \Psi \]

where \( \hbar \) = Planck constant, \( m \) = particle mass, \( \nabla^2 \) = Laplacian operator, \( \Psi \) = wave function, \( U \) = potential energy, and \( E \) = total energy. Now we consider the wave equation for one-dimensional (1D) harmonic oscillator with a force constant (K).

(1) Imose the potential energy for 1D harmonic oscillator, and write down the solvable ordinary differential equation associated with two boundary conditions.(4 points)

(2) What's the lowest energy of the 1D harmonic oscillator in terms of \( m \), \( K \), and physical constant(s) .(3 point)

(3) The O-H stretching of an alcohol molecule has a wavenumber of 3400 cm\(^{-1}\), in which the two-body harmonic oscillation occurs. What would be the wavenumber of the stretching mode, while hydrogen is substituted with deuterium(atomic mass= 2)? (3 points)