1. The parameters of the mechanical system are
   \[ M = 1000 \text{ kg}, \quad B = 10000 \text{ N/(m/sec)}, \quad K = 100000 \text{ N/m} \]

   A step force of 1000 N is applied to the mass at \( t = 0 \). The initial conditions are \( y(0) = y'(0) = 0 \). Find the damping ratio, undamped natural frequency and damped natural frequency. (15%)

2. 一個系統由以下微分方程式所描述
   \[ y''(t) + 7y'(t) + 6y(t) = 6r(t); \quad y(0) = y'(0) = 0 \]
   找出系統對輸入 \( r(t) = \sin 2t \) 的穩態響應。 (15%)

3. 對以下的特性方程式，畫出根軌跡圖。 (20%)
   \[ 1 + \frac{K}{s(s^2 + 6s + 10)} = 0; \quad K \geq 0 \]

4. In the Figure shown below with \( G(s) = 1/[(s+1)(s+7)] \):

   (1) Find the lowest value of \( K \) that will minimize the settling time. (5%)
   (2) Find \( K \) and the corresponding steady-state error for a unit step to obtain a system damping ratio of about 0.7. (5%)
   (3) Compare the settling times of parts (1) and (2). Which is the best to minimize rise time, and why? (5%)
5. For a unity feedback motor position servo with loop gain function

\[ G(s) = \frac{K}{s(0.25s + 1)(0.1s + 1)} \]

(1) Sketch the loci of the closed-loop system poles for varying \( K \). (8%)  
(2) Find \( K \) for a damping ratio 0.5 of the dominating pair. (7%)

6. Referring to the Figure shown below, where \( k \) is the spring constant, \( b \) is the damping coefficient. A man drops a steel ball of mass \( m \) onto the center of mass \( M \) from a height \( d \) and catches it on the first bounce. Assuming that the system is initially at rest, what is the motion of mass \( M \) after it is hit by the steel ball? Assume that the impact is perfectly elastic. In addition, assume that the numerical values of \( M, m, b, k \), and \( d \) are given as \( M=1 \text{ kg}, m=0.1 \text{ kg}, b=4 \text{ N-s/m}, k=125 \text{ N/m}, \) and \( d=1 \text{ m} \). The displacement \( x \) of mass \( M \) is measured from the equilibrium position before the ball hits it. The initial conditions are \( x(0) = 0 \) and \( \dot{x}(0) = 0 \). (10%)

7. Consider the spring-mass system as shown below, \( k \) is the spring constant. The system is initially at rest, or \( x(0) = 0 \) and \( \dot{x}(0) = 0 \). At \( t=0 \) a force \( p(t) = P \cos \omega t \) is applied to the mass \( m \). When the numerical values of \( m, k, P, \) and \( \omega \) are given as \( m=1 \text{ kg}, k=100 \text{ N/m}, P=50 \text{ N}, \) and \( \omega=5 \text{ rad/sec} \), find the solution \( x(t) \). (10%)