1. Solve the following differential equations:

   (1) \( xy'' + 2y = 4y^2 \) \( \quad \) \( (10\%) \)
   (2) \( x^2 y'' - 2xy' + 2y = 2\ln(x) - 1 \) \( \quad \) \( (10\%) \)

2. Use Laplace Transform method to solve the following equations.

   (15\%)
   \[
   \begin{align*}
   x'' - 2x' + 3y' + 2y &= 4 \\
   - x' + 2y' + 3y &= 0 \\
   \text{where} \quad x(0) &= x'(0) = y(0) = 0
   \end{align*}
   \]

3. Find the first five nonzero terms of the Maclaurin power series solution of the initial value problem. (15\%)

   \[
   y'' + 3y' + 2y = x \quad \text{where} \quad y(0) = 0 \quad \text{and} \quad y'(0) = 1
   \]
4. Consider the matrix $A$.

$$A = \begin{pmatrix} -1 & 0 \\ 1 & -5 \end{pmatrix}. \quad (15\%)$$

(1) Find the Eigenvalues and Eigenvectors. \hspace{1cm} (8\%)
(2) Compute $A^{18}$. \hspace{1cm} (7\%)

5. Use the Fourier transform to solve

$$y'' + 6y' + 5y = \delta(t-3). \quad (15\%)$$

6. Solve the telegraph equation

$$\frac{\partial^{2}u}{\partial t^{2}} + A \frac{\partial u}{\partial t} + Bu = c^{2} \frac{\partial^{2}u}{\partial x^{2}}$$

for $0 < x < L$, $t > 0$.

$A$ and $B$ are positive constants. The boundary conditions are

$$u(0,t) = u(L,t) = 0 \quad \text{for} \quad t \geq 0,$$

and the initial conditions are

$$u(x,0) = f(x), \quad \frac{\partial u}{\partial t}(x,0) = 0 \quad \text{for} \quad 0 \leq x \leq L.$$ 

Assume that $A^2 L^2 < 4(B L^2 + c^2 \pi^2)$. \hspace{1cm} (20\%)