(Show every step of your solution.)

1. (8 points) Determine the number of integer solutions of \( x_1 + x_2 + x_3 < 16 \), where \( x_i \geq 2 \) for \( 3 \geq i \geq 1 \).

2. (8 points) Determine the value of positive integer \( k \) such that \( (7k^3 - 21k^2 + k - 3) \) is a prime number.

3. (8 points) Determine the number of strings in \( A^3 \) and \( A^4 \), where the alphabet set \( A \) is defined as \( A = \{v, x, y, z\} \).

4. (8 points) If \( (Z_{15}, *) \) is a cyclic group, find all generators of \( (Z_{15}, *) \).

5. (8 points) Let \( B = \{a, b, c, d, e\} \). Determine the number of relations on \( B \) that are reflexive and symmetric.

6. (10 points) Given \( k \) matrices \( A_1, A_2, \ldots, A_k \), assume the matrix-multiplication-chain \( A_1 \times A_2 \times \ldots \times A_k \) follows the association law.
   (1) (5 points) Write down the recurrence relation for counting the number of ways for calculating the matrix-multiplication-chain \( A_1 \times A_2 \times \ldots \times A_k \).
   (2) (5 points) Solve your derived recurrence relation.

7. (20 points)
   (a) (8 points) Find a basis that spans the plane \( x + 2y + z = 0 \).
   (b) (7 points) Find the matrix that represents the projection onto the plane \( x + 2y + z = 0 \).
   (c) (5 points) Find the matrix that represents the reflection of through the plane \( ax + by + cz = 0 \), where \( (a, b, c) \) is a unit vector.

8. (8 points) Mike chooses either pizza or sandwich for lunch. If he chooses pizza for lunch one day, there is a \( \frac{2}{3} \) chance that he chooses pizza again the next day. If he chooses sandwich for lunch one day, there is a \( \frac{3}{7} \) chance that he chooses pizza the next day. Over the long term, what is the chance that Mike chooses pizza for lunch on any given day?

9. (10 points) Find a curve of the form \( y = a + (\frac{b}{x}) \) that best fits the data set \( \{(2, 3), (1, 4), (4, 1)\} \).

10. (12 points) Let \( B_1 = \{(1, 1), (1, -1)\} \) and \( B_2 = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\} \) be bases of \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \) respectively, and \( A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 1 & 0 \end{pmatrix} \) be the matrix of a linear transformation \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) with respect to \( B_1 \) and \( B_2 \). Find the matrix of \( T \) with respect to the standard bases of \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \).