1. This problem is concerned with the sampling of continuous-time signals. Let us consider two baseband signals \( x(t) \) and \( y(t) \). The signal \( x(t) \) is band-limited to 23 KHz, and the signal \( y(t) \) is band-limited to 40 KHz.
   (a). (5%) Is sampling a linear operation?
   (b). (5%) Find the Nyquist sampling rate for \( x(3t) + y(t) \).
   (c). (5%) Find the Nyquist sampling rate for \( x(t)y(t) \).

2. Let \( u(t) \) denote the unit-step function, which is defined by
   \[
   u(t) = \begin{cases} 
   1, & \text{if } t \geq 0, \\
   0, & \text{if } t < 0.
   \end{cases}
   \]
   Then, \( p(t) = A(u(t) - u(t - T)) \), where \( A \) is a positive constant, is a pulse of magnitude \( A \) and width \( T \) (located in \( 0 < t < 7 \)). Consider a non-return-to-zero (NRZ) waveform
   \[
   x(t) = \sum_{n=-\infty}^{\infty} d_n \times p(t - nT),
   \]
   where \( d_n \) is a binary random variable whose probability distribution is
   \[
   \text{Prob}(d_n = 1) = \text{Prob}(d_n = -1) = \frac{1}{2}.
   \]
   (a). (5%) Find the autocorrelation function of \( x(t) \).
   (b). (5%) Find the power spectral density of \( x(t) \).

3. Let \( \text{Prob}(E) \) denote the probability of an event \( E \). The \( Q \) function is defined to be \( Q(x) = \text{Prob}(Z > x) \), where \( Z \) is the standard Gaussian random variable (i.e., a Gaussian random variable with a mean of 0, and a variance of 1).
   (a). (5%) Let \( Q'(x) \) denote the derivative of \( Q(x) \). Then, \( Q'(-1) = ? \)
   (b). (5%) Let \( X \) be a Gaussian random variable with a mean of \( \mu \), and a variance of \( \sigma^2 \). It can be shown
   \[
   \text{that } \text{Prob}(X > t) = Q(\alpha t + \beta). \text{ Then, } \alpha = ?, \text{ and } \beta = ? \text{. (Please express your answer in terms of } \mu \text{ and } \sigma.)
   \]
   (c). (5%) The \text{erfc} function is defined by
   \[
   \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-u^2} du.
   \]
   It can be shown that \( Q(x) \) and \( \text{erfc}(x) \) are related to each other by
   \[
   Q(x) = A \times \text{erfc}(B x + C).
   \]
   Then, \( A = ?, \ B = ?, \ \text{and } C = ? \).

4. In BPSK (binary phase shift keying) signal transmission, data bit 0 and data bit 1 are, respectively, mapped into waveforms \( s_0(t) = A \cos(2\pi f_t t) \) and \( s_1(t) = A \cos(2\pi f_t t + \pi) \), for \( 0 < t < T \). Notice that one data...
bit occupies a time duration of $T$.

(a). (5%) What is the null-to-null bandwidth, which is equal to the width of the main lobe of the signal power spectrum, consumed by this BPSK transmission? Please express your answer in terms of $T$.

(b). (5%) Can BPSK be demodulated non-coherently? Please also explain the difference between non-coherent demodulation and coherent demodulation.

5. (10%) Suppose $x(t)$ has the Fourier transform

$$X(f)$$

Find $y(t)$ in terms of $x(t)$ if $y(t)$ has the Fourier transform

$$Y(f)$$

6. For a amplifier, the relation of the input signal $u(t)$ and output signal $w(t)$ is given by

$$w(t) = 10u(t) + 5u^3(t).$$

If we use this amplifier to amplify a frequency modulation (FM) signal $u(t)$, please answer following questions.

(a) (5%) Compute the distortion signal in $w(t)$.

(b) (5%) How to remove the distortion signal in $w(t)$?

7. (10%) Suppose that the pulse amplitude modulation (PAM) signal $s(t) = \sum_{n=1}^{N} a_n g(t-nT)$ passes through the multipath channels with the impulse response $h(t) = \delta(t) + 0.5\delta(t-2T)$. Please compute the matched filter which can maximize the ratio of the signal power to noise power at the receiver.

8. (5%) Suppose that a communication system uses 5MHz bandwidth to transmit data through the additive white Gaussian noise (AWGN) channel. If 256-QAM modulation is used, please find the maximum transmission rate (bits/sec) if there is no intersymbol interference.

9. If $X$ and $Y$ are independently and identically Gaussian distributed with mean 0 and variance $\sigma^2$, please answer following questions.

(a) (5%) Compute the probability density function of the random variable $Z = X^2 + Y^2$.

(b) (10%) Prove the random variable $Z = aX + bY$ is Gaussian distributed if both $a$ and $b$ are constants.