1. Consider the problem of locating a new machine to an existing layout consisting of five machines. These machines are located at the following coordinates: \((3, 1), (2, 3), (3, 4), (4, 3), (5, 2)\). Let the coordinates of the new machine be \((x_1, x_2)\). Formulate a linear program to minimize the sum of the street distances from the new machine to the five existing machines. (20%)

[Hint: Street distance from \((x_1, x_2)\) to the machine located at \((2, 5)\) is \(|x_1 - 2| + |x_2 - 5|\).]

2. An RRT (round robin tournament) is being held between 10 football teams. Each team plays every other team exactly 6 times and every play ends with one of the two participating teams as the winner and the other the loser. We are given a vector \(\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{10})\) of nonnegative integers with the claim that \(\alpha_i = \text{total number of wins in the RRT for team } i, \text{ for } i = 1, 2, \ldots, 10\). It is required to check whether this claim can be correct.
   a) Formulate a network flow model for checking it. (20%)
   b) Given \(\alpha = (20, 12, 31, 38, 35, 13, 32, 40, 10, 39)\), use the model developed in part (a) to check whether the claim is correct. (10%)

3. Consider a Markov chain with states 0, 1, 2, 3, 4. Suppose \(P_{i,i+1} = 1\); and suppose that when the chain is in state \(i, i > 0\), the next state is equally likely to be any of the states 0, 1, \ldots, \(i - 1\). Find the limiting probabilities of this Markov chain. (15%)

4. Potential customers arrive at a single-server station in accordance with a Poisson process with rate \(\lambda\). However, if the arrival finds \(n\) customers already in the station, then she/he will enter the system with probability \(a_n\). Assuming an exponential service rate \(\mu\), set this up as a birth and death process and determine the birth and death rate. (15%)

5. For the M/M/1 queue (let the arrival rate be \(\lambda\), and service rate be \(\mu\)), compute
   a) the expected number of arrivals during a service period. (10%)
   b) the probability that no customers arriving during a service period. (10%)