1. The quantization of a signal $x(t)$ is considered in this problem. Assume that $x(t)$ can be modeled as a zero-mean Gaussian random variable. In the quantizer design, it is desired that the distortion, which is measured by the mean squared errors, be minimized.

(a). (5%) Is quantization a linear operation?

(b). (5%) Suppose that one of the quantization regions is $[a, b]$ (i.e. the interval from $a$ to $b$), where $0 < a < b$. Let the optimal quantization level in this region be denoted as $q$. Then, which of the statements below is true: (i). $q < (a+b)/2$, (ii). $q = (a+b)/2$, (iii). $q > (a+b)/2$?

(c). (5%) Let $r$ and $t$ be two adjacent quantization levels. Assume that $0 < r < t$. Let the optimal boundary point that divides $(r, t)$ (i.e. the interval from $r$ to $t$) into two regions be denoted as $s$. Then, which of the statements below is true: (i). $s < (r+t)/2$, (ii). $s = (r+t)/2$, (iii). $s > (r+t)/2$?

2. (10%) Assume that $X$ is a Rayleigh-distributed random variable with a parameter value of 1. In other words, the probability density function (pdf) of $X$ is

$$f(x) = \begin{cases} x \times \exp\left(-\frac{x^2}{2}\right), & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases}$$

Let a random variable $Y$ be defined as $Y = X^2$. Then, the pdf of $Y$ is in the form of

$$g(y) = \begin{cases} a \times y^b \times \exp(c \times y^d), & \text{if } y \geq 0, \\ 0, & \text{if } y < 0. \end{cases}$$

Find the values of $a$, $b$, $c$, and $d$.

3. In BPSK modulation, data bit 0 and data bit 1 are mapped into waveforms $s_0(t) = A \cos(2\pi f_c t)$ and $s_1(t) = -s_0(t)$, for $0 < t < T$ ($T$ is one bit duration), respectively. Assume that the signals are transmitted over an AWGN channel. At the receiving end, a correlator is adopted for demodulation. Let the correlator output be denoted as $z$. Then, the likelihood function for $z$ is

$$f(z|s_0) = \frac{1}{\sigma \sqrt{2\pi}} \times \exp\left(-\frac{(z - A)^2}{2\sigma^2}\right), \quad \text{if } s_0 \text{ was sent}$$

$$f(z|s_1) = \frac{1}{\sigma \sqrt{2\pi}} \times \exp\left(-\frac{(z + A)^2}{2\sigma^2}\right), \quad \text{if } s_1 \text{ was sent}$$

Where $A$ and $\sigma$ are some positive numbers.

(a). (5%) Assume that data bit 0 and data bit 1 are equally likely to be sent by the transmitter. If $z = 0.13$ is observed, should the decision be "bit 0" or "bit 1"?

(b). (5%) Assume that data bit 0 and data bit 1 are equally likely to be sent by the transmitter. Then, based on the MAP (maximum a-posteriori) criterion, what is the threshold value for $z$ beyond which a decision of "bit 0" will be made?

(c). (5%) Assume that the probability that the transmitter sends out data bit 0 is 0.7 (then, of course, the probability that the transmitter sends out data bit 1 is 0.3). Moreover, for simplicity, let us assume that $\sigma = 1$ and $A = 1$. Then, based on the MAP criterion, what is the threshold value for $z$ beyond which a decision of "bit 0" will be made?
4. Consider a (7,3) linear block code whose generator matrix is
\[
\begin{bmatrix}
1 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 1 \\
\end{bmatrix}
\]
(a) (5%) What is the minimum distance of the code?
(b) (5%) Perform decoding for the received sequence [0 0 1 1 1 1 1]. In other words, what is the decoded codeword, and what is the decoded message word?

5. (10%) Consider a superheterodyne receiver which receives signals with bandwidth 2MHz. Assume there is a filter prior to mixer. What is the range of input signals will be received if \( f_m = 6 \text{MHz} \)?

6. Consider the random variable \( Y \) given by
\[
Y = \frac{1}{N} \sum_{i=1}^{N} X_i
\]
where \( X_i \) for \( 1 \leq i \leq N \) are statistically independently and identically distributed with the distribution
\[
p(X) = \begin{cases} 
p, & X = 1 \\ 1-p, & X = 0 \end{cases}
\]
Please answer the following questions:
(a) (5%) Determine the characteristic function of \( Y \).
(b) (5%) Find the probability density function of \( Y \) as \( N \to +\infty \).

7. Three signals are transmitted over AWGN channel with noise power spectral density \( N_0/2 \). Three signals are
\[
s_1(t) = \begin{cases} 
1, & 0 \leq t \leq 2 \\
0, & \text{elsewhere}
\end{cases}
\]
\[
s_1(t) = -s_2(t) = \begin{cases} 
-1, & 0 \leq t \leq 1 \\
1, & 1 < t \leq 2 \\
0, & \text{elsewhere}
\end{cases}
\]
Please answer the following questions:
(a) (10%) Find a orthogonal basis for the signal space.
(b) (5%) Draw the signal constellation.
(c) (5%) Determine the optimal decision region.
(d) (10%) Find the minimum error probability.