1. Consider the following second order constant-coefficient ordinary differential equation:
\[ a\ddot{y} + b\dot{y} + cy = 3, \quad y(0) = 1 \text{ and } \dot{y}(0) = 2 \]
(a) If \( a = 1, b = 3, c = 2 \), find the solution \( y(t) = ? \) \hspace{1cm} (15%)
(b) If \( a = 1, c = 2 \), and \( b \) is a tunable parameter, what will be the condition for \( b \) in order to get an oscillatory solution with decaying amplitude? \hspace{1cm} (5%)

2. Use the method of Laplace transform to solve the following initial value problem:
\[ \dot{y} + y = \frac{1}{a} [U(t) - U(t-a)], \quad y(0) = 0 \text{ and } \dot{y}(0) = 0, \]
where \( U(t) \) is the unit step function defined as follows:
\[ U(t) = \begin{cases} 0 & t < 0, \\ 1 & t \geq 0. \end{cases} \]
What will be the solution if \( a \to 0 \)? \hspace{1cm} (20%)

3. Let \( A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \), answer the following questions:
(a) Diagonalize the matrix \( A \). \hspace{1cm} (10%)
(b) Solve the following system of ordinary differential equations
\[ \frac{d}{dt} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = A \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} \]
subject to the following initial conditions:
\[ \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \]
\hspace{1cm} (10%)

4. By evaluating both sides of the equation, verify Stokes' theorem for the vector field \( \vec{F} = 4xz\vec{i} - 2x\vec{j} + 2x\vec{k} \) over the plane of intersection of the cylinder \( x^2 + y^2 \leq 1 \) and the plane \( z = y + 1 \). \hspace{1cm} (20%)

5. Starting from separation of variables, solve the boundary-value problem
\[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 \leq r \leq 2, \quad 0 \leq z \leq 1 \]
\[ u(r,0) = 0, \quad u(r,1) = 1, \quad u(2,z) = 0. \] \hspace{1cm} (20%)