Improved Transmission Strategies for Cognitive Radio Under the Coexistence Constraint

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Abstract—In this work, we consider an interference-mitigation based cognitive radio system where a secondary transmitter is to communicate with its corresponding receiver without affecting the communication between a primary transmitter-receiver pair. In this case, the secondary transmitter must satisfy a coexistence constraint which requires that no rate degradation occurs at the primary user (PU), even when the latter utilizes only a single-user decoder. To achieve a non-zero rate of the secondary user (SU) under this constraint, Jovicic and Viswanath previously proposed a scheme (referred to as the JV scheme) that utilizes relaying by the secondary transmitter to overcome the interference caused by the simultaneous transmission of the SU’s message. In this case, the interference caused by the signals corresponding to PU’s message at the secondary receiver (from both the direct and the relay links) is mitigated by employing dirty paper coding (DPC) at the secondary transmitter. However, the interference caused by the SU’s message at the primary receiver is not eliminated in this case and, thus, will limit the power (and, hence, the rate) that can be used by the secondary transmitter to transmit its own message. In our work, the use of clean relaying by the secondary transmitter and/or receiver (i.e., relaying without simultaneous transmission of SU’s own message) is proposed to improve the quality of the relayed signal and, thereby, increases the rate achievable by the SU. Two improved transmission schemes are proposed: (i) clean relaying by secondary transmitter (CT) and (ii) clean relaying by secondary transmitter and receiver (CTR). The CT scheme utilizes DPC to mitigate interference at the secondary receiver whereas the CTR scheme utilizes coding for multiple access channels with common messages to enable decoding of both PU’s and SU’s messages at the secondary receiver. The CT scheme can be viewed as a generalization of the JV scheme and, therefore, performs at least as well as the latter. The CTR scheme, on the other hand, is shown to outperform the CT scheme in terms of the multiplexing gain achievable under full channel state information at the transmitter (CSIT) and in terms of the rate achievable with statistical CSIT. Numerical simulations are provided to illustrate these advantages.

Index Terms—Cognitive radio, Coexistence constraint, Clean relaying, Transmitter relaying, Receiver relaying, Partial CSIT, Dirty paper coding, Gelfand-Pinsker coding.

I. INTRODUCTION

Efficient spectrum usage is essential to satisfying the increasing demands for high data rate services. However, due to this reason, many research efforts have been devoted to studies on the so-called cognitive radio (CR) technology [1], which enables unlicensed (or secondary) users to dynamically sense and locate unused spectrum segments and to communicate via these unused spectrum segments without causing harmful interference on licensed (or primary) users. Most CR systems initially proposed in the literature, e.g., [2] and references within, adopt the interference avoidance based approach, which requires secondary users (SUs) to vacate the spectrum once the existence of primary user’s (PU’s) signals have been sensed. This requires fast and accurate sensing of spectrum holes, and may not yield the most efficient spectrum utilization since only one user can access the spectrum at any given time. In this work, we consider instead an interference mitigation [3], [4] based CR system, where SUs attempt to mitigate their interference at PUs even when transmitting simultaneously in the same spectrum. In this case, the secondary transmitters must satisfy the coexistence constraint [4], which requires that their transmissions do not cause any rate degradation at PUs even when the latter employ only single-user decoders.

To achieve a non-zero rate for an SU under the coexistence constraint, Jovicic and Viswanath proposed in [4] a scheme (hereon referred to as the JV scheme) that utilizes relaying by the secondary transmitter to overcome the interference caused by the simultaneous transmission of SU’s message. For the SU, the interference caused by the received signals corresponding to PU’s message at the secondary receiver (which is received from both the direct and the relay links) is mitigated by employing dirty paper coding (DPC) [4], [5] at the secondary transmitter. This transmission scheme has been shown to be capacity-achieving when the channel gain between secondary transmitter and primary receiver is smaller.

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than that between the secondary transmitter and receiver [4]. However, even though DPC mitigates interference at the secondary receiver, the interference caused by the SU’s message at the primary receiver is not eliminated in this case and, thus, the power (and, hence, the rate) used by the secondary transmitter to transmit its own message must be limited in order to satisfy the coexistence constraint. Moreover, when employing DPC, the secondary transmitter must be able to reliably decode PU’s message and full channel state information at the transmitter (CSIT) must be available. The former may not be possible if the channel between the primary and the secondary transmitters is not sufficiently reliable and the latter is difficult to obtain in fast fading environments.

The main contribution of this work is to propose a new transmission strategy that utilizes clean relaying by the secondary transmitter and/or receiver (i.e., relaying without simultaneous transmission of SU’s own message) to improve the quality of the relayed signal and, thereby, increase the rate achievable by the SU. Two transmission schemes are proposed: (i) clean relaying by secondary transmitter (CT) and (ii) clean relaying by secondary transmitter and receiver (CTR). The CT scheme is a generalization of the DPC-based scheme proposed in [4] (i.e., the JV scheme) with an additional phase that is used to perform clean relaying by the secondary transmitter. The CTR scheme, on the other hand, enables clean relaying by both the secondary transmitter and receiver, and utilizes coding for multiple access channels (MAC) with common messages [6]. In cases with full CSIT, the CTR scheme may not always achieve a higher rate compared to the CT scheme under moderate signal-to-noise ratio (SNR) values (e.g., when the channel between the primary transmitter and the secondary receiver is not sufficiently reliable), but does always achieve a higher multiplexing gain as the SNR goes to infinity. When only statistical CSIT is available, we observe that CT does not provide any rate advantages in fast Rayleigh fading environments (compared to naive single-user encoding and decoding schemes) and, thus, CTR should be used instead. In fact, CTR even requires less complexity than both CT and JV schemes [4]. Simulation results are provided to demonstrate the efficiency of the proposed CT and CTR schemes compared to that of the JV scheme [4].

The techniques adopted in this paper are similar to the concept of cooperation studied in the literature on interference channels, e.g., in [7], [8]. However, the coexistence constraint was not considered in those works since it is only essential to cognitive radio applications. Yet, this constraint can make the problem considerably more challenging. Since previous works did not consider the coexistence constraint, the capacity results obtained in the literature on interference channels are generally less restrictive and, therefore, can serve as performance outer bounds for our setting. Moreover, most previous works, e.g., [3], [4], [7]–[9], considered only the full CSIT scenario whereas our work also considers the statistical CSIT case. Compared to our previous work in [9], where only a preliminary study of the CTR scheme has been provided, this work includes additionally the proposition of the CT scheme as well as complete analysis of the multiplexing gains of both schemes.

The remainder of this paper is organized as follows. In Section II, a general description of the system model is provided. In Sections III and IV, the CT and CTR schemes are proposed and their achievable rate and multiplexing gain performances under full CSIT are derived. The cases with only statistical CSIT in fast Rayleigh fading channels are examined in Section V. The performance of the proposed schemes are demonstrated through numerical simulations in Section VI and the paper is concluded in Section VII.

Notations: The superscript $(\cdot)^H$ denotes the complex conjugate transpose. A block-diagonal matrix with diagonal entries $A_1, \ldots, A_k$ is denoted by $\text{diag}(A_1, \ldots, A_k)$ and the determinant of a square matrix $A$ is denoted by $|A|$. The function $(x)^+ = \max(x, 0)$ and $(x)^- = \max(-x, 0)$ otherwise. $I_n$ denotes the identity matrix with dimension $n$ and $E[\cdot]$ denotes the expectation.

II. SYSTEM MODEL

Let us consider a basic four-node cognitive radio channel as shown in Fig. 1, where nodes 1 and 4 represent the primary transmitter and receiver while nodes 2 and 3 represent the secondary transmitter and receiver, respectively. The signals transmitted by nodes 1, 2, and 3 are denoted by $X_1(t)$, $X_2(t)$, and $X_3(t)$, respectively. The transmitted signals are assumed to satisfy the average power constraints given by

$$\frac{1}{n} \sum_{i=1}^{n} |X_i(t)|^2 \leq \bar{P}_i, \quad \text{for } i = 1, 2, 3,$$

where $n$ is the code length and $t$ is the index of code symbol. The signals received at nodes 2, 3, and 4 at symbol-$t$ are given by

$$Y_2(t) = h_{12}(t)X_1(t) + Z_2(t),$$
$$Y_3(t) = h_{13}(t)X_1(t) + h_{23}(t)X_2(t) + Z_3(t),$$
$$Y_4(t) = h_{14}(t)X_1(t) + h_{24}(t)X_2(t) + h_{34}(t)X_3(t) + Z_4(t),$$

respectively, where $h_{ij}(t)$ is the channel coefficient between nodes $i$ and $j$, and $Z_i(t)$ is the additive white Gaussian noise process at node $i$. We assume that $\{Z_i(t)\}_{i=1}^{n}$ are independent and identically distributed (i.i.d.) circularly-symmetric complex Gaussian, i.e., $Z_i(t) \sim \mathcal{CN}(0, 1)$. Note that all nodes are half-duplex.
In this paper, we examine the performance of our proposed scheme under both slow and fast fading scenarios. For both scenarios, we assume that all receivers have full knowledge of its receiving channels. In the slow fading scenario, we assume that the channel remains constant throughout the transmission of a codeword, i.e., \( h_{ij}(t) = h_{ij} = \alpha_{ij} e^{j\theta_{ij}}, \) for \( t = 1, \ldots, n, \) where \( \theta_{ij} \) is the channel phase. When transmitting, we also assume that node 1 has perfect knowledge of \( h_{14}, \) whereas nodes 2 and 3 have perfect knowledge of all channel coefficients. The CSIT at node 2 can be obtained by overhearing and utilizing the channel feedback sent from the receiver to the transmitter, as proposed in [4]. The CSIT at node 1 and 3 can be obtained similarly. In the fast fading scenario, we assume that the channel coefficients \( \{h_{ij}\}_{i=1}^{n} \) are i.i.d. \( \mathcal{CN}(0, \sigma_{c}^2) \) and that only the channel statistics are known at the transmitters. Specifically, we assume that node 1 knows only the statistics of \( h_{14}(t) \) and that nodes 2 and 3 know the statistics of all channels (except those receiving on, in which case, full CSIT is available). Similar to [10, Sec. II], we also assume that SU is able to obtain knowledge of the PU’s target rate by eavesdropping on the PU’s control channel.

In cognitive radio systems, PUs should not be required to change their transmitting and receiving strategies due to the presence of SUs. Therefore, as in [4], the primary transmitter and receiver, i.e., nodes 1 and 4, are restricted to using only single-user encoders and decoders, i.e., those that can be used to reliably transmit messages over point-to-point channels [4]. Let \( E_1^n \) be a rate \( R_1 \) encoder at the primary transmitter that encodes the PU’s message \( w_1 \in \{1, \ldots, 2^{nR_1}\} \) into a length-\( n \) codeword \( X_1^n \triangleq [X_1(1), \ldots, X_1(n)] \in \mathcal{X}_1^n, \) where \( X_1 \) is the channel input alphabet for the primary transmitter. Assume, without loss of generality, that the maximum-likelihood decoder is employed at the primary receiver [11]. A single-user achievable rate is defined as follows.

**Definition 1:** A rate \( R_1 \) is single-user achievable if there exists a sequence of \( (2^{nR_1}, n) \)-codes that can be decoded with a single-user decoder at the primary receiver with vanishing error probability as \( n \to \infty. \)

On the other hand, since SUs are trying to access PU’s channel, it is possibly beneficial for SUs to encode their messages utilizing knowledge of PU’s codebook (when it can be reliably obtained). The availability of PU’s codebook at secondary transmitter and receiver is for enabling the SU to decode the PU’s message. Together with the received signals, the secondary transmitter and receiver can decode the PU’s messages according to the PU’s codebook, respectively. Now we can define SU’s achievable rate as follows. Suppose that \( w_2 \in \{1, \ldots, 2^{nR_2}\} \) is the message to be transmitted by the secondary transmitter and let \( E_2^n \) be a rate \( R_2 \) encoder that encodes the message pair \( (w_1, w_2) \) into a length-\( n \) codeword \( X_2^n \triangleq [X_2(1), \ldots, X_2(n)] \in \mathcal{X}_2^n, \) where \( X_2 \) is the channel input alphabet for the secondary transmitter. We introduce the following coexistence constraint, which is one of the main differences between the considered model and the common interference channels considered in [7] [8].

**Definition 2:** The coexistence constraint is a constraint that requires the SU to ensure that, under its own transmission, PU’s required rate \( R_1 \) is still single-user achievable as per Definition 1.

The SU’s achievable rate is defined as

**Definition 3:** A rate \( R_2 \) is achievable by the SU if there exists a sequence of \( (2^{nR_2}, n) \)-codes with encoders \( E_2^n \) that can be decoded with vanishing error probability at the secondary receiver as \( n \to \infty \) while satisfying the coexistence constraint.

In the following sections, we denote PU’s target rate as \( R_T \) and select it as

\[
R_T = C(|h_{14}|^2 \bar{P}_1),
\]

i.e., PU’s maximum achievable rate in the absence of SUs, where \( C(x) \triangleq \log(1 + x) \). In addition, improved transmission strategies based on the clean relaying technique are proposed to increase the rate achievable by the SU under the coexistence constraint.

### III. Clean Relaying by Secondary Transmitter (CT) under Full CSIT

In this section, we describe the proposed CT scheme and derive its performance under full CSIT. To improve upon [4], the CT scheme employs an additional clean-relaying phase to improve the quality of relaying by the secondary transmitter. As opposed to [4], our analysis also takes into account the portion of the codeword used for the secondary transmitter to reliably decode PU’s message when computing the achievable rates. The signaling in each phase of the CT scheme is described as follows and is illustrated in Fig. 2(a).

Recall that \( X_1(t) \) and \( X_2(t) \) are the signals transmitted by nodes 1 and 2, respectively, at symbol-\( t \). The CT scheme employs three phases of transmission that occupy a total of \( n \) channel uses, as described below.

**Phase 1:** In the first \( n_1 \) channel uses, i.e., for \( 1 \leq t \leq n_1 \), only node 1 transmits while node 2 remains silent (i.e., \( X_2(t) = 0 \)) and attempts to decode the PU’s message \( w_1 \) from the overheard signal \( Y_2(t) \). The duration of Phase 1 (i.e., \( n_1 \)) is chosen adaptively to ensure that node 2 successfully decodes the PU’s message. (If this cannot be done within \( n \) channel uses, then Phases 2 and 3 cannot be employed and the SU’s achievable rate \( R_2 \) will be zero in this case.)

*The celebrated Shannon’s random codebook [12] is adopted in our paper, which guarantees the existence of codebook with decodable \( w_1 \) in Phase 1 as in the seminal papers [13] [14]. The design of the practical codebook capable of such early decoding is a hot research topic and beyond the scope of this paper. However, we still give a brief discussion to show that practical design is feasible. First, one may use the classical Reed-Solomon code. The key is to treat the non-received coded symbols as erasures. As long as the number of correct received code symbols (non-erased symbols) is large enough, the message can be successfully decoded without knowing the entire codewords [15]. Moreover, advanced codes such as Fountain and Raptor codes can also ensure the early decoding. More details about the capacities of such code can be found in [16]."
Phase 2: Once node 2 successfully decodes message \( w_1 \) in Phase 1, it will help forward the second part of node 1’s codeword, i.e., \( \{X_1(t)\}_{t=n_1+1}^{n_1+n_2} \), in Phase 2 while simultaneously transmitting its own message \( w_2 \). This can be done by assuming knowledge of node 1’s codebook at node 2. Specifically, in the \( n_2 \) channel uses of Phase 2 (i.e., for \( t = n_1 + 1, \ldots, n_1 + n_2 \)), node 2 will transmit

\[
X_2(t) = U_2^D(t) + \sqrt{\alpha_1 P_2^2 / P_1} e^{j(\theta_1 t - \theta_2 t)} X_1(t),
\]

where \( \alpha_1 \) is the relay ratio that is adjusted to eventually maintain rate \( R_T \) on the primary link, \( P_1 \) and \( P_2^2 \) are the powers of \( X_1(t) \) and \( X_2(t) \), respectively, in Phase 2, and \( \{U_2^D(t)\}_{t=n_1+n_2+1}^{n_1+n_2} \) is the DPC encoded signal [5] with power \((1 - \alpha_1) P_2^2 \). Here, \( \{U_2^D(t)\}_{t=n_1+n_2+1}^{n_1+n_2} \) encodes node 2’s message \( w_2 \) using the known interference seen by node 3, i.e., \( \{(h_{13} + h_{23} \sqrt{\alpha_1 P_2^2 / P_1} e^{j(\theta_1 t - \theta_2 t)}) X_1(t)\}_{t=n_1+n_2+1}^{n_1+n_2} \), as the side information at the transmitter.

Phase 3: In the remaining \( n_3 = n - n_1 - n_2 \) channel uses, node 2 performs clean relaying by transmitting the third part of node 1’s codeword without super-imposing its own message. The signal transmitted by node 2 can be written as

\[
X_2(t) = \sqrt{P_2^2 / P_1} e^{j(\theta_1 t - \theta_2 t)} X_1(t), \quad t = n_1 + n_2 + 1, \ldots, n,
\]

where \( \{X_1(t)\}_{t=n_1+n_2+1}^{n} \) is the third part of the codeword transmitted by node 1. As will be shown in Theorem 1, node 2 can successfully decode \( w_1 \) in Phase 1 as long as \( \eta_1 \) is long enough, where \( \eta_1 = n_i / n \) is the portion of the codeword used in Phase \( i \). With the knowledge of \( w_1 \) and the codebook of the primary user, in Phase 2, node 2 then knows the primary user’s transmitted codeword \( X_1(t) \) and can use it in Phase 2 and 3 as described above.

Since node 1 transmits in all three phases and node 2 transmits only in Phases 2 and 3, the transmit powers \( P_1 \), \( P_2^2 \), and \( P_2^2 \) can be set as

\[
P_1 = \bar{P}_1 \quad \text{and} \quad \eta_2 P_2^2 + \eta_3 P_2^2 = \bar{P}_2,
\]

where \( \bar{P}_1 \) and \( \bar{P}_2 \) are the average power constraints given in (1). Note that the signal-to-interference-plus-noise ratio (SINR) at node 4 changes in each phase. Hence, the following lemma is necessary to compute PU’s single-user achievable rate throughout the three phases.

Lemma 1: Consider a Gaussian scalar channel where the signals received over \( n \) channel uses can be expressed as \( Y^n = H^n X^n + Z^n \). Here, \( H^n \) is an \( n \times n \) diagonal matrix with the \( n \) channel coefficients on its diagonal and \( Z^n \) is the Gaussian noise vector. By assuming that \( H^n \) is known at the receiver, then rate \( R \) will be single-user achievable if

\[
R < \lim_{n \to \infty} \frac{1}{n} \log \left( \frac{\|H^n K_{X^n}(H^n)^+ K_{Z^n}\|}{\|K_{Z^n}\|} \right),
\]

where \( K_{Z^n} \) and \( K_{X^n} \) are the covariance matrices of \( Z^n \) and \( X^n \), respectively.

The lemma can be proved following the procedures given in the proof of Theorem 5 in [17]. With (5), the rate achievable
by the SU with the CT scheme can be derived as follows. The proof is given in Appendix A. Notice that, with this choice of $R_T$ in (5), no information can be transmitted by the SU if node 2 cannot decode or does not help forward PU’s message.

**Theorem 1:** For the CT scheme assume that full CSIT is available at node 2. The rate $R_2$ is achievable by the SU if

$$R_2 \leq \max_{\eta_1, \eta_2, \alpha_1, P_2} \eta_2 C \left( |h_{23}|^2 (1 - \alpha_1) P_2^s \right),$$

subject to $R_T \leq \eta_1 C (|h_{12}|^2 P_1)$ (i.e., PU’s message is decodable at node 2) and the coexistence constraint

$$R_T \leq \eta_1 C (|h_{12}|^2 P_1) + \eta_2 C \left( \left( |h_{14}| \sqrt{P_1} + |h_{24}| \sqrt{\alpha_1 P_2^s} \right)^2 \right) + \eta_3 C \left( \left( |h_{14}| \sqrt{P_1} + |h_{24}| \sqrt{P_2 - \eta_2 P_2^s} \right)^2 \right).$$

(10)

In fact, the optimal value of $\eta_1$ is given by $\eta_1^* = R_T / C (|h_{12}|^2 P_1)$, which is the minimum portion of the code length required for node 2 to decode PU’s message.

This result takes into consideration the latency required for node 2 to decode PU’s message, which is in contrast to the scheme [4] with unrealistic assumption that the SU can know $w_1$ without Phase 1 (where $\eta_1$ is set as 0). Moreover, in [4], the clean relaying phase was not employed (i.e., $\eta_3 = 0$) and, thus, much of node 2’s available power must be used to forward PU’s message in order to interfere the reception caused by $U_2(t)$. This reduces the effectiveness of relaying by node 2.

Following the definition in [18] [19], the multiplexing gain (or degree of freedom) of SU for a fixed value of $P_1$ is defined as

$$m_2 = \lim_{P_s \to \infty} \frac{R_2}{\log P_s},$$

(12)

where $P_s$ is the average transmission power of SU. Note that $P_s$ is equal to the average transmission power of node 2 in the CT scheme, i.e., $P_s = P_2$ ($P_s$ is equal to the total average transmission power of nodes 2 and 3 in the CTR scheme (c.f. Section IV.).) From (5), and let $(\eta_1^*, \eta_2^*, \alpha_1^*, P_2^s)$ be the optimal solution given by Theorem 1. Notice that, since $P_1$ is fixed, $R_T$ is finite and, thus, the coexistence constraint can be satisfied with finite $P_2^s$ in Phase 3. In this case, by the constraint that $\eta_2 P_2^s + \eta_3 P_2^s = P_s$, the power $P_2^s$ used in Phase 2 approaches $P_2^s / \eta_2$ as $P_s \to \infty$. Hence, the multiplexing gain achieved with the CT scheme can be upper-bounded as

$$m_2 = \lim_{P_s \to \infty} \frac{\eta_2 C \left( |h_{23}|^2 (1 - \alpha_1) P_2^s \right)}{\log P_s} = \lim_{P_s \to \infty} \eta_2^* = \lim_{P_s \to \infty} \eta_2^* \leq \lim_{P_s \to \infty} \left[ \eta_2^* + (1 - \eta_1^* - \eta_2^*) \right] \leq \left( 1 - \frac{C(|h_{14}|^2 P_1)}{C(|h_{12}|^2 P_1)} \right)^+, \quad (13)$$

where the last inequality is obtained by replacing $\eta_1^*$ with $R_T / C (|h_{12}|^2 P_1)$ as given in Theorem 1. The upper bound obtained here becomes tight as $P_s$ goes to infinity since, in this case, the coexistence constraint can be satisfied with an arbitrarily small Phase 3 duration (i.e., $1 - \eta_1^* - \eta_2^* \to 0$ as $P_s \to \infty$). This shows that the multiplexing gain is limited by the portion of the codelength node 2 can successfully decode in Phase 1, i.e., $C(|h_{14}|^2 P_1) / C(|h_{12}|^2 P_1)$. Motivated by this observation, in the following section we propose the CTR scheme that improves upon this by allowing the secondary receiver to share the load of relaying PU’s message.

**IV. CLEAN RELAYING BY SECONDARY TRANSMITTER AND RECEIVER (CTR) UNDER FULL CSIT**

In this section, we propose a scheme that allows both the secondary transmitter and receiver to help and perform clean relaying. In this scheme, node 2 need not always be able to successfully decode node 1’s message. In fact, whoever is able to successfully decode PU’s message after Phases 1 and 2 will be allowed to perform the clean relaying in Phase 3. Since node 2’s transmission may also be used to help node 3 decode PU’s message, we choose to use coding schemes designed for MAC with common message [6] instead of DPC. Note that this coding scheme has lower complexity compared to DPC [20]. Since different coding schemes are used, the CTR scheme cannot be viewed as a generalization of the CT scheme and, in fact, does not always perform better. However, we will show that, with full CSIT, the multiplexing gain of CTR is no less than that of CT (and its special case in [4]). The advantage is even more evident in systems with only statistical channel uses, only node 2 can successfully decode node 1’s message.

**Phase 1:** In the first $n_1$ channel uses, only node 1 transmits while both nodes 2 and 3 attempt to decode the message $w_1$. Unlike in the CT scheme, $n_1$ in the CTR scheme need not be long enough to ensure successful decoding at node 2.

**Phase 2:** In the next $n_2$ channel uses, node 2 can choose to either transmit only its own message or transmit a superposition of signals that convey both PU’s and SU’s messages. The signal transmitted by node 2 can be expressed generically as

$$X_2(t) = U_2^M(t) + \sqrt{\alpha_1 P_2^s / P_1} e^{j\theta_1} X_1(t),$$

(14)
for $t = n_1 + 1, \ldots, n_1 + n_2$, where $\{X_1(t)\}_{t=n_1+1}^{n_1+n_2}$ is the second part of node 1’s codeword and $\{U_2^M(t)\}_{t=n_1+1}^{n_1+n_2}$ is the signal that encodes SU’s message $w_2$. We assume that $U_2^M(t)$ encodes $w_2$ independently of $X_1(t)$. Here, $\alpha_1$ is the relay ratio and $\theta_1$ is the relay phase (which was chosen as $\theta_{14} - \theta_{24}$ in the CT scheme). The power transmitted by node 2 in Phase 2 is $P_2'$ and the power of $U_2^M(t)$ is $(1 - \alpha_1) P_2'$. If node 2 is not able to decode $w_1$ from Phase 1, it will transmit only its own message, in which case, $\alpha_1 = 0$ and node 3 will see standard MAC. On the other hand, if node 2 is able to successfully decode $w_1$, it may be beneficial to choose $\alpha_1 > 0$, in which case, node 3 will see an asymmetric MAC with a common message. The signaling scheme in (14) was shown to be optimal for the Gaussian MAC with common message in [6]. Finally, node 3 tries to decode both $w_1$ and $w_2$ based on its received signals in both phases.

**Phase 3:** In the remaining $n-n_1-n_2$ channel uses, signals by clean relaying are transmitted by either node 2 or node 3 or both, depending on whoever was able to successfully decode $w_1$ in the previous two phases and the realizations of $h_{24}$ and $h_{34}$. The signals transmitted by nodes 2 and 3 can be written as

$$X_2(t) = \sqrt{P_2''/P_1} e^{i(\theta_{14} - \theta_{24})} X_1(t),$$

$$X_3(t) = \sqrt{P_3'/P_1} e^{i(\theta_{13} - \theta_{34})} X_1(t),$$

for $t = n_1 + n_2 + 1, \ldots, n$, where $P_2''$ and $P_3$ are the powers transmitted by nodes 2 and 3 in Phase 3.

Similarly, let $\eta_1 = n_i/n$ be the portion of the code length used for the transmission of Phase $i$. To satisfy the power constraint in (1), we set the transmit powers $P_1, P_2', P_2''$, and $P_3$ such that

$$P_1 = \tilde{P}_1, \quad \eta_2 P_2' + \eta_3 P_2'' = \tilde{P}_2, \quad \text{and} \quad \eta_3 P_3 = \tilde{P}_3,$$

if both nodes 2 and 3 are able to decode PU’s message. If either node 2 or node 3 is not able to decode, we shall set $P_2'' = 0$ or $P_3 = 0$, respectively. The following lemma from [21] is needed to compute the achievable rate of CTR. This can be viewed as an extension of the results in Lemma 1 to the MAC setting.

**Lemma 2:** Let $X^n$ and $U^n$ be two independent blocks of length-$n$ Gaussian symbols transmitted through a scalar MAC and let $\mathbf{H}_x^n X^n + \mathbf{H}_u^n U^n + \mathbf{Z}^n$ be the received signals, where $\mathbf{H}_x^n$ and $\mathbf{H}_u^n$ are $n \times n$ diagonal matrices with $n$ corresponding channel coefficients on its diagonal and $\mathbf{Z}^n$ is the Gaussian
noise with covariance matrix $K_Z^n$. By assuming that $H^n_2$ and $H^n_u$ are known at the receiver, the rate pair $(R_1, R_2)$ is achievable if

$$R_1 \leq \lim_{n \to \infty} \frac{1}{n} \log \left( \frac{H^n_2 K_{X^n} (H^n_u)^H + K^n_{Z^n}}{|K^n_{Z^n}|} \right),$$

$$R_2 \leq \lim_{n \to \infty} \frac{1}{n} \log \left( \frac{|H^n_u K_{U^n} (H^n_u)^H + K^n_{Z^n}|}{|K^n_{Z^n}|} \right),$$

$$R_1 + R_2 \leq \lim_{n \to \infty} \frac{1}{n} \log \left( \frac{|H^n_2 K_{X^n} (H^n_2)^H + H^n_u K_{U^n} (H^n_u)^H + K^n_{Z^n}|}{|K^n_{Z^n}|} \right),$$

where $K_{X^n}$ and $K_{U^n}$ are the covariance matrices of $X^n$ and $U^n$, respectively.

By combining the results in [6] with Lemmas 1 and 2, one can show that SU’s codebooks can be chosen such that successful decoding of both messages is ensured at node 3 and that the coexistence constraint is met at node 4. When deriving the achievable rate, we consider the same setting as (5). As shown in Table I, we consider three cases: both nodes 2 and 3 can successfully decode, only node 2 can successfully decode, and only node 3 can successfully decode. From (5), the coexistence constraint can not be satisfied unless $R_2 = 0$ for the case where neither node 2 nor node 3 can decode. This case is trivial and excluded. Since the three cases are mutually exclusive, SU’s achievable rate can be found as the maximum rate among them. Note that in addition to the coexistence constraint, Table I also lists the constrained parameter sets and variable sets under which the three cases occur, respectively. Then the achievable rate of the CTR scheme can be found by taking the maximum among the rates of the three cases as in the following.

**Theorem 2:** For the CTR scheme, assume that full CSIT is available at nodes 2 and 3, and that $R_T = C(|h_{14}|^2 P_1)$. Given $P_1$, $P_2'$, and $P_3$, the coexistence constraint is satisfied if

$$R_T \leq \eta_1 C(|h_{14}|^2 P_1) + \eta_2 C \left( \frac{|h_{14} + h_{24}| \sqrt{\alpha_1 P_1}}{1 + |h_{24}|^2 (1 - \alpha_1) P_2} \right)^2 \right)$$

$$+ \eta_3 C \left( \frac{|h_{14}| \sqrt{P_1} + |h_{24}| \sqrt{P_2'} + |h_{34}| \sqrt{P_3}}{1} \right)^2 \right).$$

(17)

In this case, the rate $R_2$ is achievable by the SU if $R_2 \leq \max\{R_{2,1}, R_{2,2}, R_{2,3}\}$, where $R_{2,1}$, $R_{2,2}$ and $R_{2,3}$ given in (18), (19), and (20) (shown in the next page), are the achievable rates for the three cases, respectively, with the corresponding constrained parameter sets $S_k$, for $k = 1, 2, 3$, specified in Table I.

The proof is given in Appendix B. Note that the coding scheme used in CTR is different from that in CT and, thus, does not always guarantee rate advantage over CT (at least under the full CSIT assumption). Indeed, if the same relaying ratio $\alpha_1$ is used for both CT and CTR, the coding scheme of CT performs better than that of CTR (c.f. (10) and (18)). However, due to the new channel for clean relaying from node 3 to node 4, CTR may have smaller $\alpha_1$ than CT, and thus may have rate advantages. Moreover, unlike CT, even if node 2 is not able to decode message $w_1$, the coexistence constraint may still be met with CTR by having node 3 relay the message. This allows the CTR to have longer portion of the code length for transmitting SU’s own message and to achieve a higher multiplexing gain as shown in the following corollary. The multiplexing gain is defined as in (12), where the average transmission power of the SU is now equal to

$$\bar{P}_s = \eta_2 P_2' + \eta_3 P_2'' + \eta_3 P_3 = \bar{P}_2 + \bar{P}_3. \quad (21)$$

**Corollary 1:** With full CSIT at nodes 2 and 3, the multiplexing gain $m_2$ is achievable by the CTR scheme, where

$$m_2 = \max \left\{ \mathcal{M}_m \left( 1 - \frac{C(|h_{14}|^2 P_1)}{C(|h_{12}|^2 P_1)} \right)^+ \right\}, \quad (22)$$

with $m = a_{\text{si}}$, when $|h_{14}| < |h_{12}|$, and $m = 0$, otherwise, for any

$$a \in \left[ \frac{C(|h_{14}|^2 ar{P}_1)}{C(|h_{12}|^2 ar{P}_1)}, 1 \right].$$

The proof is given in Appendix C. In fact, according to Appendix C, the multiplexing gains

$$(1 - C(|h_{14}|^2 P_1)/C(|h_{12}|^2 P_1))^+$$

and $m$ correspond to the CTR scheme using only node 2 and only node 3 to help relay PU’s message, respectively. Also according to Appendix C, increasing $a$ requires larger SNR for the generalized multiplexing gain (GMG) [22] of practical systems to approach the performance predicted by the MG analysis. Comparing (22) with (13), we observe that, in the case with full CSIT, the multiplexing gain achieved by the CTR scheme is larger (or at least no less) than that of CT (and its special case in [4]). In the next section, we will investigate the performance of the CTR and CT in fast Rayleigh fading channels with only statistical CSIT. The CTR is even more promising in this setting.

**Remark:** Fundamentally, in a more general information theoretic point of view, the difference between CT and CTR is the new channel between nodes 3 and 4. In our work we only propose the new CTR signaling to utilize this new channel. The optimal signaling scheme is definitely of interest but is a challenging open problem. In fact, this scheme involves the relaying of PU’s signals whereas the capacity of relay channels is still unknown. The capacity upper bounds in [4] can be extended to our channel model by introducing Phase 1 to decode PU’s message. However, finding the optimal scheme that achieves this upper bound is difficult. In this work, we focus on deriving an achievable scheme to improve upon that in [4], which is currently the best known scheme other than ours. Also in this paper we only consider one PU in Fig. 1. The proposed CT and CTR can be extended when there are more PUs such as the channel in [23] (or even MIMO channel using [24]).
\[ R_{2,1} \leq \max_{(\eta_1, \eta_2, \alpha_1, \theta_1, P_2^2) \in S_1} \min \left\{ \eta_2 C \left( |h_{23}|^2 (1 - \alpha_1) P_2^2 \right), \eta_1 C (|h_{13}|^2 \tilde{P}_1) + \eta_2 C \left( |h_{23}| \sqrt{\alpha_1 P_2^2 / \tilde{P}_1 e^{j \theta_1}} + |h_{13}|^2 \tilde{P}_1 + |h_{23}|^2 (1 - \alpha_1) P_2^2 - R_T \right) \right\}, \]  \tag{18}

\[ R_{2,2} \leq \max_{(\eta_1, \eta_2, \alpha_1, \theta_1, P_2^2) \in S_2} \eta_2 C \left( \frac{|h_{23}|^2 (1 - \alpha_1) P_2^2}{1 + |h_{23}| \sqrt{\alpha_1 P_2^2 / \tilde{P}_1 e^{j \theta_1}} + |h_{13}|^2 \tilde{P}_1} \right), \]  \tag{19}

\[ R_{2,3} \leq \max_{(\eta_1, \eta_2) \in S_3} \min \left\{ \eta_2 C (|h_{23}|^2 \tilde{P}_2 / \eta_2), \eta_1 C (|h_{13}|^2 \tilde{P}_1) + \eta_2 C \left( |h_{13}|^2 \tilde{P}_1 + |h_{23}|^2 \tilde{P}_2 / \eta_2 - R_T \right) \right\}, \]  \tag{20}

V. CLEAN RELAYING IN FAST RAYLEIGH FADING CHANNELS WITH STATISTICAL CSIT

In this section, we derive the ergodic rates achievable by the SU in fast Rayleigh fading environments, where only statistical CSIT is available. The achievable rate of the CTR scheme is first derived and later shown to be better than that of the CT scheme. Here, the CT and CTR schemes also follow the three phase transmission schemes described in Section IV, but with \( \theta_1, \theta_1 \) and \( \theta_2 \) set to 0 since the channel phases are unknown at the transmitter. As in the full CSIT case, SU’s achievable ergodic rate \( R_2 \) can be derived by taking the maximum among the results of three cases as shown in the following theorem. Since the channel coefficients are assumed to be i.i.d. between channel uses in the fast fading scenario, we shall use the capital letter \( H_{ij} \) to denote the random channel coefficient between node \( i \) and node \( j \). The proof is given in Appendix D.

**Theorem 3:** For the CTR scheme, suppose that statistical CSIT is available at nodes 2 and 3, and that

\[ R_T = E[C(|H_{14}|^2 \tilde{P}_1)]. \]

Given \( P_1, P_2^\prime, P_2^\prime \), and \( P_3 \), the coexistence constraint is satisfied if

\[ R_T \leq \eta_1 E[C(|H_{14}|^2 \tilde{P}_1)] + \eta_2 E \left[ C \left( \frac{|H_{14}| + \sqrt{\frac{P_1 P_2}{P_1}} |H_{24}|^2 \tilde{P}_1}{1 + |H_{24}|^2 (1 - \alpha_1) P_2^2} \right) \right] + \eta_3 E \left[ C \left( \frac{P_3}{P_1} |H_{34}|^2 \tilde{P}_1 \right) \right]. \]  \tag{23}

In this case, the ergodic rate \( R_2 \) is achievable by the SU if \( R_2 \leq \max \{ R_{2,1}, R_{2,2}, R_{2,3} \} \), where \( R_{2,1}, R_{2,2}, \) and \( R_{2,3} \) given in (24), (25), and (26) (shown in the next page), are the achievable rates for the three cases, respectively, with the constrained parameter sets \( S_k \), for \( k = 1, 2, 3 \), specified in Table I.

The optimal common message relaying ratio \( \alpha_1 \) in the above Theorem is solved in [25, Corollary 2].

For CT, we observe that, with only statistical CSIT, DPC is not able to mitigate interference. That is, the rate achievable in this case is the same as that of treating interference as noise. This observation is summarized in the following proposition with the proof given in Appendix E.

**Proposition 1:** Consider the ergodic Rayleigh fading channel with the received signal given by \( Y_4 = H_{23} X_2 + H_{13} X_1 + Z_3 \), where \( X_1 \) is the known interference at transmitter with \( E[|X_1|^2] \leq \tilde{P}_1 \) and \( X_2 \) is the DPC encoded symbol with \( E[|X_2|^2] \leq \tilde{P}_2 \). With only statistical CSIT, the rate achievable by using DPC is the same as that obtained by treating interference \( H_{13} X_1 \) as noise, which is given by

\[ E[C(|H_{23}|^2 \tilde{P}_2 / (1 + |H_{13}|^2 \tilde{P}_1))]. \]

Based on the above observation, we have the following proposition.

**Proposition 2:** Under the ergodic Rayleigh fading channel, with only statistical CSIT, the rate achievable by CTR is no less than that of CT.

**Proof:** The result is proved by showing that the rate achievable by CT under only statistical CSIT is the same as that achieved in the special case of CTR where only node 2 is able to decode PU’s message (i.e., Case 2 in Table I (b)). First, by Proposition 1, we know that the rate achievable in the CT scheme under statistical CSIT is the same as that of treating interference as noise, which is also the same as that in Case 2 of CTR, i.e., (25). Moreover, by extending (11) to the ergodic case and by substituting \( P_3 = 0 \) into (23), we can also see that the coexistence constraints of both cases are the same. Hence, the result follows. \( \blacksquare \)

Given the above results, we can then derive the multiplexing gain achievable by CTR under only statistical CSIT. The proof is similar to that of Corollary 1 and is omitted.

**Corollary 2:** With statistical CSIT, the multiplexing gain achievable by the CTR scheme is

\[ m_2 = \max \left\{ m, \left( \frac{1 - E[C(|H_{14}|^2 \tilde{P}_1)]}{E[C(|H_{12}|^2 \tilde{P}_1)]} \right) \right\}, \]  \tag{27}

where \( m = a, \) if \( E[C(|H_{14}|^2 \tilde{P}_1)] < E[C(|H_{13}|^2 \tilde{P}_1)] \), and \( m = 0, \) otherwise, for any

\[ a \in \left[ \frac{E[C(|H_{14}|^2 \tilde{P}_1)]}{E[C(|H_{13}|^2 \tilde{P}_1)]}, 1 \right]. \]

Note that the rate formulae of CT and CTR in Theorem 1-3 are both complicated optimization problems, and it is
hard to compare their performances analytically. Therefore, the MG analysis in (13), Corollary 1 and 2 is given for the performance comparison at high SNR. Moreover, one can also observe that for CTR, the maximum rate gains over CT can approach infinity from Corollary 1 and 2. Take the full CSIT case as an example, it happens when SU has high transmit power, and $|h_{13}| > |h_{14}| > |h_{12}|$. In this case, CT has zero rate since the channel between nodes 1 and 2 is too bad such that node 2 cannot decode PU’s message to meet the coexistence constraint by relaying. However, the CTR may still have positive rates due to the clean relaying at node 3. Moreover, from Corollary 1, CTR has a non-zero multiplexing gain and scales with SU’s transmit power at the high SNR regime. Thus the maximum gain can approach infinity. The statistical CSIT case follows similarly from Corollary 2.

VI. NUMERICAL RESULTS

In this section, we provide numerical results to demonstrate the efficacy of the proposed clean-relaying schemes. In the following, we will abbreviate the results of adding Phase 1 to the scheme in [4] (which is equivalent to CT with $\eta_3 = 0$) as IV. The noise variances at the receivers are set to unity, and the average transmit power of PU (i.e., $P_1$ in (1)) is set to 20 dB. We assume that the SU in both CT (including IV) and CTR have the same average transmit power $P_s$, which can be computed according to (8) ($P_s = P_2$) and (21), respectively. We define the transmit SNR, which is used in the figures, as the ratio of the transmit power to the noise variance. Note that since our system is a multi-user network, it is more convenient to use the transmit SNR instead of the conventional receive SNR while the former is usually larger than the latter. We also set the PU’s transmission rate $R_T$ as the capacity of the primary link in the absence of SU, i.e., $C(|h_{14}|^2P_1)$ in the full CSIT case and $E[C(|h_{14}|^2P_1)]$ in the case with only statistical CSIT.

We first compare the rates measured in bits per channel use (bpcu) in the full CSIT case. The channel gains of each figure are listed in Table II where the unit of the phases is radian. In Fig. 3, we can see that with large enough $|h_{34}|$ as specified in Table II, clean relaying by node 3 allows CTR to achieve the best performance. When $|h_{34}|$ is sufficiently smaller than $|h_{24}|$, as shown in Fig. 4, CTR may prefer clean relaying by node 2 over node 3 in the considered SNR regions, that is, $P_2' > P_3 = 0$. Without the aids of additional clean relaying from node 3, the CT outperforms CTR in Fig. 4. However, we can find that the rates of CT and CTR converge when the SNR increases. This is because in the considered channel setting, the first argument of the upper bound of $R_{31}$ in (18) is selected at high SNR. Then the rate formulae of CTR and CT are the same (c.f. (18) and (10)). Moreover, the considered $|h_{34}|$ is sufficiently smaller than $|h_{24}|$ in this figure and thus CTR uses node 2 to relay in Phase 3 ($P_3 = 0$). The $\eta_3$ of CTR will be close to that of CT. This result comes from that with $P_3 = 0$, the coexistence constraints for CT and CTR in (11)
TABLE II
CHANNEL GAINS AND VARIANCES IN (2)-(4) USED IN THE SIMULATIONS (FULL AND STATISTICAL CSIT)

<table>
<thead>
<tr>
<th>Figure</th>
<th>$h_{14}$</th>
<th>$h_{24}$</th>
<th>$h_{34}$</th>
<th>$h_{13}$</th>
<th>$h_{23}$</th>
<th>$h_{12}$</th>
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</thead>
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<tr>
<td>3</td>
<td>$0.36e^{-1.6}_j$</td>
<td>$0.45e^{-1.6}_j$</td>
<td>$0.96e^{-4.1}_j$</td>
<td>$0.74e^{-1.19}_j$</td>
<td>$0.25e^{-0.69}_j$</td>
<td>$0.24e^{-1.89}_j$</td>
</tr>
<tr>
<td>4</td>
<td>$0.22e^{-1.6}_j$</td>
<td>$0.92e^{-0.45}_j$</td>
<td>$0.32e^{-2.16}_j$</td>
<td>$0.52e^{-0.95}_j$</td>
<td>$0.19e^{-0.22}_j$</td>
<td>$0.19e^{-1.4}_j$</td>
</tr>
<tr>
<td>5</td>
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<td>varying $</td>
<td>h_{24}</td>
<td>, \theta_{24} = \pi/4$</td>
<td>varying $</td>
<td>h_{34}</td>
</tr>
<tr>
<td>6</td>
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<td>$0.19e^{-2.09}_j$</td>
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<td>$0.24e^{-1.84}_j$</td>
</tr>
<tr>
<td>7</td>
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</tr>
<tr>
<td>9</td>
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<td>$0.74e^{-1.19}_j$</td>
<td>$0.96e^{-0.87}_j$</td>
<td>$0.24e^{-1.84}_j$</td>
<td>$0.24e^{-1.84}_j$</td>
</tr>
<tr>
<td>10</td>
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<td>$0.19e^{-1.4}_j$</td>
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<tr>
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<td>$0.95e^{-1.95}_j$</td>
<td>varying $</td>
<td>h_{34}</td>
<td>, \theta_{34} = \pi/4$</td>
<td>varying $</td>
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<table>
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<th>$\sigma^2_{23}$</th>
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</tr>
<tr>
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<td>0.12</td>
<td>0.87</td>
<td>0.92</td>
<td>0.96</td>
<td>1</td>
</tr>
</tbody>
</table>

and (17) will be approximately the same except that there is an additional relaying phase term $\theta_1$ for CT in (17). However, as long as the transmit power of SU increases, the effect of $\theta_1$ decreases. Then $\alpha_1$, and thus the rate of CT will converge to those of CT in Fig. 4. In both Figs. 3 and 4, we can see that both CT and CTR outperform the JV scheme. Next, we show how SU’s rate changes with $|h_{24}|$ in Fig. 5. We can see that there are two regions. In Region 1, i.e., $|h_{24}| \leq |h_{23}|$, CT and JV coincide, which is consistent with [4]. That is, when relaying from node 3 is prohibited, JV is proved to be optimal in this region. However, we need to emphasize that the JV scheme shown in our simulations takes into consideration the non-zero decoding time at node 2 in Phase 1 and, therefore, is different from the scheme in [4]. The fact that the CT scheme reduces to the JV scheme when $|h_{24}(t)| \leq |h_{23}(t)|$, can also be shown theoretically by using the bounding techniques given in the converse proof of [4, Theorem 3.1].

In Region 2, $|h_{24}| > |h_{23}|$, JV wastes much power on relaying since SU produces large interference at node 4. Due to clean relaying, CT outperforms JV. Also in Region 2, when $|h_{24}|$ is sufficiently smaller than $|h_{34}| = 0.3$, CTR outperforms CT since CTR can use a much better relaying path from node 3 than that of CT in Phase 3. However, when $|h_{24}|$ increases, CT performs better and eventually better than CTR. Similar to the high SNR discussions for Fig. 4, it results from the fact that when $|h_{24}|$ becomes closer or larger than $|h_{34}|$, CTR tends to use node 2 to perform clean relaying in Phase 3. In this case, $P_3 = 0$ for CTR, and the relaying ratio $\alpha_1$ for CT will be smaller than that of CTR since the coexistence constraints in (11) and (17) will be approximately the same except that there is an additional relaying phase term $\theta_1$ for CT in (17). Furthermore, since CT achieves the interference-free rate as $\eta_1 C([h_{14}]^2 P_4) / C([h_{13}]^2 P_4)$, which will be no less than the achievable rate of CTR under the same $\alpha_1$. Therefore, we can conclude that for large enough $|h_{24}|$, CT outperforms CTR. In the simulation, we scan all $\eta_1 \epsilon C([h_{14}]^2 P_4) / C([h_{13}]^2 P_4)$ and find that the optimal $\eta_1 = \eta_1^* = C([h_{14}]^2 P_4) / C([h_{12}]^2 P_1)$, which is the smallest feasible value for $\eta_1$. Note that in Corollary 1, the second argument of the $\max\{\cdot\}$ in (22) is $1 - \eta_1$, which can be maximized by minimizing $\eta_1$, too.

We also provide the numerical results as Fig. 5 but with varying $|h_{12}|$ and $|h_{13}|$ in Fig. 6 and 7, respectively. In Fig. 6, we can find that when $|h_{12}| = 0.3$, which is smaller than $|h_{14}| = 0.36$, node 2 cannot decode PU’s message in Phase 1, thus CT and JV have zero rates. But CTR can still use pure receiver relay from node 3 to have a non-zero rate ($R_2 = R_{23}$ in (20)). Also the rate in (20) does not change with $|h_{12}|$. When $|h_{12}| > |h_{14}|$, node 2 can decode PU’s message in Phase 1, thus CT and JV have positive rates. However, in the considered channel setting, CTR outperforms CT and JV since $|h_{13}|$ and $|h_{34}|$ are sufficiently large in Table II. In Fig. 7, we observe that there are three regions according to $|h_{13}|$. In the region where $|h_{13}| \geq 0.7$, node 3 can apply successive decoding [12] such that SU can operate at rate where the interference from the PU is completely canceled out (first arguments of $R_{21}$ in (18)). With the clean relaying
Fig. 6. Comparison of SU’s rate performances under full CSIT for different values of $|h_{12}|$ (c.f. Table II).

Fig. 7. Comparison of SU’s rate performances under full CSIT for different values of $|h_{13}|$ (c.f. Table II).

Fig. 8. Comparison of SU’s rate performances (c.f. Table II).

Fig. 9. Comparison of SU’s rate performances in fast Rayleigh fading channels with statistical CSIT. The channel variances are listed in Table II.

from node 3, compared with the CT, the CTR can use smaller $\alpha_1$ for relaying in Phase 2, and having higher rate. Also the SU’s rate $R_2$ does not change with $|h_{13}|$ in this region. In the second region where $0.2 \leq |h_{13}| < 0.7$, node 3 must jointly decode the messages of PU and SU. That is, the second argument of $R_{21}$ in (18) is selected, and $R_2$ decreases with decreasing $|h_{13}|$ since in our channel setting, the optimal $\theta_1$ will make $|h_{23}\sqrt{\alpha_1 P'_2/P_1 e^{j\theta_1} + h_{13}}|^2$ increase with $|h_{13}|$. It is because the smaller $|h_{13}|$ may cause node 2 to increase $\alpha_1$ on relaying PU’s signal to make $w_1$ decodable at node 3. And thus less power is used to transmit SU’s own signal. Note that when $|h_{13}|$ is too small, CTR may be worse than CT and JV. Although there is additional clean relaying from node 3 for CTR, such rate advantage may not compensate the loss from increasing $\alpha_1$. Finally, in the region where $|h_{13}| \leq 0.2$, SU chooses node 2 instead of node 3 to relay, and node 3 treats interference from PU as noise while decoding ($R_{22}$ in (19) is selected). Note that in this region, $R_2$ increases with decreasing $|h_{13}|$ since the interference from node 1 is smaller. As for CT and JV, rates $R_2$ do not change with $|h_{13}|$. This fact can be easily observed from Theorem 1. Finally we consider the case that $|h_{14}| > |h_{34}|$ in Fig. 8. In this case, $|h_{34}|$ is significantly smaller than $|h_{14}|$ and $|h_{24}|$ is larger than $|h_{14}|$, thus CTR tends to choose node 2 to relay. As the discussions for Region 2 of Fig. 5, without the diversity provided by the relaying path $h_{34}$, both CTR and CT will have similar relaying ratios $\alpha_1$. And CT will be better than the CTR since CT can always achieve the rate as if the interference of PU is absent at node 3.

Next we consider the rate performance in the fast Rayleigh fading channels with statistical CSIT. The channel variance of each link is listed in Table II. As shown in Fig. 9, the CTR outperforms the CT and JV, which is consistent with Proposition 2 in Section V. When the SU’s transmit SNR is low, the CT (also JV) can only support very low rate as shown in Fig. 9. This is because that the PU’s transmit SNR is set to 20 dB, then the interference at node 3 is relatively large.
And in Fig. 10 and 11 we set a lower bound for $\eta_{R}$. We use the GMG [22] of the SU, which is defined as the clean relaying. We can avoid this problem. But CT still outperforms JV owing to the rate performance. The MAC decoder of CTR at node 3 interference from the PU is noise, which significantly degrades to Proposition 1, the SU of CT (also JV) performs as if the $m$ and 11 with channels specified in Table II, respectively. Note that the $m$ in Corollary 1 is an asymptotic value for the GMG. And in Fig. 10 and 11 we set a lower bound for $\eta_3$ as 0.01 when $\eta_3 \neq 0$. That means $m$ will be upper-bounded by 1-

for the SU when the SU’s transmit SNR is low. According to Proposition 1, the SU of CT (also JV) performs as if the interference from the PU is noise, which significantly degrades the rate performance. The MAC decoder of CTR at node 3 can avoid this problem. But CT still outperforms JV owing to the clean relaying.

Finally, we compare the multiplexing gains of CT and CTR. We use the GMG [22] of the SU, which is defined as $R_2/\log P_s$, as the performance metric for finite SNR. As $P_s$ approaches infinity, the GMG becomes the multiplexing gain defined in (12). We first show the full CSIT cases in Fig. 10 and 11 with channels specified in Table II, respectively. Note that the $m$ in Corollary 1 is an asymptotic value for the GMG. And in Fig. 10 and 11 we set a lower bound for $\eta_3$ as 0.01 when $\eta_3 \neq 0$. That means $m$ will be upper-bounded by 1-

0.01=0.99. The reason we choose this value is to maintain the achievable MG $m$ large enough which is close to 1, i.e., the supremum of MG in Corollary 1, while being able to make the GMG converge towards $m$ within the considered SNR range.

We then use the multiplexing gain in (22) with $m=0.99$ as the GMG upper bound in Fig. 10 and 11. With large $|h_{34}|$ as in Table II, the GMG advantages of the CTR over the JV can be seen from Fig. 10. Since $|h_{14}| < |h_{13}|$ in this simulation, according to discussions under Corollary 1, the CTR with only node 3 relaying has larger GMG than those of the CT and JV when $\eta_3$ is small and the SNR is large enough. Also when the SNR increases, the GMG of the CTR will approach the upper bound (22). In Fig. 11, we show the case with small $|h_{34}|$. The CT performs the best while the CTR performs the worst. From our simulation result, we can observe that the second argument of $\min\{\cdot\}$ in (18) is selected in this setting, and CT outperforms CTR. However, as predicted by Corollary 1, even though the CTR has the worst GMG, it will approach those of CT and JV as the SNR increases. In Fig. 12 (shown in the next page), we compare the GMG of the three schemes with different $|h_{34}|$. It can be found that under the channel setting with $P_s=50$ dB, CTR always have better GMG than CT independent of $|h_{34}|$ except when $|h_{34}| < 0.1$. The behavior of $|h_{34}| < 0.1$ can be explained by the discussions for Fig. 11. The GMG results for the fading channels with statistical CSIT are shown in Fig. 13 (shown in the next page). The GMG upper bound is computed from Corollary 2 with $m=0.99$ as in Fig. 10. According to the discussions for Fig. 9, the CT and JV always have worse GMG than that of the CTR according to Proposition 1. We can also observe that the GMGs of both JV and CT decrease with increasing $P_2$. The reason is that from Proposition 1, we know that the GMGs of both JV and CT converge to zero when $P_2 \to \infty$. The main difference between CT and JV is the different $\alpha_1$’s. Due to CT has additional Phase 3 to provide clean relay, it has smaller $\alpha_1$ than JV, which makes the rate of CT larger than that of JV. Thus CT has higher GMG than JV.
transmit signals in Phases 1 to 3, the received signal at node utilizes a DPC encoding scheme at node transmitter and receiver (CTR). The CT scheme follows \[4\] and version schemes were proposed: (i) clean relaying by secondary order to satisfy the coexistence constraint. Two new transmissions simultaneously with PU but must also help relay PU’s signal in this work, where SUs are allowed to transmit and derive the single user achievable rate at node 4.

\[ H^n = \text{diag}\left( \begin{pmatrix} h_{14}I_{n_1}, \left| h_{14} \right| + \left| h_{24} \right| \sqrt{\frac{\alpha_1 P_2^u}{P_1}} e^{j\theta_{14}} I_{n_2}, \left| h_{14} \right| + \left| h_{24} \right| \sqrt{\frac{P_2^u}{P_1}} e^{j\theta_{14}} I_{n_3} \end{pmatrix} \right) \]

and

\[ K_{Z^n} = \text{diag}\left( I_{n_1}, (1 + \left| h_{24} \right|^2 (1 - \alpha_1) P_2^u) I_{n_2}, I_{n_3} \right). \]

Notice that the DPC encoded $U_2^D$ \[5\], which is Gaussian distributed with variance $(1 - \alpha_1) P_2^u$, is independent of $X_1$ and is treated as noise at node 4. The single-user achievable rate given in (11) then follows from Lemma 1. On the other hand, by using $(h_{13} + h_{23} \sqrt{\frac{\alpha_1 P_2^u}{P_1}} e^{j(\theta_{13} - \theta_{23})}) X_1(t)$ as the side-information at node 2 in Phase 2, the interference caused by $X_1(t)$ at node 3 are mitigated and, hence, SU’s achievable rate is given in (10). Moreover, we can show that the optimal $\eta_1$ is given by its minimum possible value $\eta_1^* = R_T / C(\left| h_{12} \right|^2 P_1)$. To show this, suppose that the optimal solution is given by $(\eta_1^*, \eta_2^*, \alpha_1^*, P_2^{\alpha_1^*})$ where $\eta_1^* > \eta_1^\dagger$. In this case, we can take $\eta_1$ such that $\eta_1^* < \eta_1^\dagger$, which will increase the right-hand-side of (11) since the third term in (11) is always greater than the first term. Then, we can either find $\alpha_1 < \alpha_1^\dagger$ or (if $\alpha_1^\dagger = 0$) find $\eta_2 > \eta_2^\dagger$ satisfying (11) such that the achievable rate is increased. This contradicts the assumption that $(\eta_1^*, \eta_2^*, \alpha_1^*, P_2^{\alpha_1^*})$ is optimal and hence the optimal $\eta_1$ is given by $\eta_1^* = R_T / C(\left| h_{12} \right|^2 P_1)$.

B. Proof of Theorem 2

The proof is obtained by analyzing separately 3 possible scenarios: the case where both nodes 2 and 3 can successfully decode PU’s message (Case 1), the case where only node 2 can successfully decode (Case 2), and the case where only node 3 can successfully decode (Case 3). The achievable rates for those cases are denoted by $R_{2,1}$, $R_{2,2}$, and $R_{2,3}$, respectively.

Case 1: Consider the case where nodes 2 and 3 are both able to decode PU’s message. In this case, node 3 sees an asymmetric MAC with common message $w_1$ received from both nodes 1 and 2 and private message $w_2$ received from node 2. From \[6\], we know that the optimal distribution of input signals $X_1$ and $U_2$, corresponding to the messages $w_1$ and $w_2$, respectively, should be independent and Gaussian with variance $P_1$ and $(1 - \alpha_1) P_2^u$, respectively. Here, PU’s and SU’s codebooks are generated according to the distributions of $X_1$ and $U_2$ with rates $R_T$ and $R_{2,1}$, respectively. As mentioned in \[6\], the achievable rate region of MAC with common messages can be reduced to an equivalent MAC without common messages \[12\]. Since only node 1 transmits in Phase 1 whereas both nodes 1 and 2 transmit in Phase 2, the effective SNRs of PU’s and SU’s messages vary across different phases and, therefore, Lemma 2 is invoked to compute the achievable rate. From the
signal model in (2)-(4), we can take
\[ K_{\nu} = (1 - \alpha_1)P_2' I_n, \]
\[ K_{X} = \bar{P}_1 I_n, \]
\[ H_{n} = \text{diag}(0 \cdot I_{n_1}, h_{23} I_{n_2}, 0 \cdot I_{n_3}), \]
\[ H_{x} = \text{diag}(h_{13} I_{n_1}, (h_{23} \sqrt{\alpha_1 P_2'/\bar{P}_1 e^{j\theta_1} + h_{13}}) I_{n_2}, 0 \cdot I_{n_3}), \]
\[ K_{z} = I_n. \quad (29) \]
Thus, by Lemma 2, the correct decoding of \((w_1, w_2)\) at node 3 is subject to the rate constraints
\[ R_T \leq \eta_1 \cdot C(\|h_{13}\|^2 \bar{P}_1) + \eta_2 \cdot C \left( h_{23} \sqrt{\alpha_1 P_2'/\bar{P}_1 e^{j\theta_1} + h_{13}} \right)^2 \bar{P}_1, \]
\[ R_{2,1} \leq \eta_2 \cdot C (|h_{23}|^2 (1 - \alpha_1) P_2'), \]
\[ R_{2,1} + R_T \leq \eta_1 \cdot C(|h_{13}|^2 \bar{P}_1) + \eta_2 \cdot C \left( h_{23} \sqrt{\alpha_1 P_2'/\bar{P}_1 e^{j\theta_1} + h_{13}} \right)^2 \bar{P}_1 + |h_{23}|^2 (1 - \alpha_1) P_2', \quad (32) \]
Eqs. (31) and (32) lead to the inequality on the achievable rate \(R_{2,1}\) given in (18) whereas eq. (30) yields the constraint on the correct decoding of \(w_1\) at node 3 as shown in Table I. The constraint on the correct decoding of \(w_1\) at node 2 is given by the channel coding theorem [12]. Moreover, the condition in (17) can be derived by invoking Lemma 1 with (14) and (15) and by following the steps in Appendix A.

Case 2: Now we consider the case where only node 2 is able to decode PU’s message. In this case, node 3 still sees an asymmetric MAC with a common message as in Case 1 and, thus, the constraint resulting from the inability to decode at node 3 is given by
\[ R_T > \eta_1 C (|h_{13}|^2 \bar{P}_1) + \eta_2 C \left( h_{23} \sqrt{\alpha_1 P_2'/\bar{P}_1 e^{j\theta_1} + h_{13}} \right)^2 \bar{P}_1. \]
Since node 3 is not able to correctly decode \(w_1\), it simply treats PU’s signal \(X_1\) as noise when decoding \(w_2\). Hence, given \(\eta_1, \eta_2, \alpha_1, \theta_1, \) and \(P_2'\), the achievable rate \(R_2\) must satisfy
\[ R_{2,2} \leq \eta_2 \cdot C \left( \frac{|h_{23}|^2 (1 - \alpha_1) P_2'}{1 + |h_{23}|^2 \sqrt{\alpha_1 P_2'/\bar{P}_1 e^{j\theta_1} + h_{13}} \bar{P}_1} \right)^2. \quad (33) \]
The achievable rate can then be maximized over the parameters \(\eta_1, \eta_2, \alpha_1, \theta_1, \) and \(P_2'\). Moreover, since node 3 is not able to help relay PU’s message, the coexistence constraint is given as in (17) with \(\bar{P}_3 = 0\).

Case 3: Here, we consider the case where only node 3 is able to decode PU’s message. In this case, node 3 faces a conventional MAC without common messages but with varying SNRs across different phases. This can be viewed as a special case of Case 1 and, thus, the rate constraints for \(R_{2,3}\) and \(R_T\) follow similarly from (30)-(32) with \(\alpha_1 = 0, P_2' = 0,\) and, thus, \(P_2' = \bar{P}_3/\eta_2\). The constraint \(\eta_1 C(|h_{12}|^2 \bar{P}_1) < R_T\) in Table I means that Phase 1 is not long enough for node 2 to decode \(w_1\). The coexistence constraint is also given as in (17) with \(\alpha_1 = 0\) and \(P_2' = \bar{P}_3/\eta_2\).

C. Proof of Corollary 1

To derive the achievable multiplexing gain, we consider two achievable transmission strategies: the case where only node 2 (c.f. Case 1) and the case where only node 3 (c.f. Case 3) helps relay PU’s message, regardless of whether the other node is able to transmit. Note that these cases are selected only for the tractability of the derivation, and the optimality for multiplexing gain (MG) may not be guaranteed. However, we can show that the achievable MG of CTR from these suboptimal selections is larger than that of CT. We show in the following that the achievable multiplexing gains in these two cases are
\[ \lim_{\bar{P}_s \to \infty} \frac{R_{2,1}}{\log \bar{P}_s} = \left( 1 - \frac{C(|h_{14}|^2 \bar{P}_1)}{C(|h_{12}|^2 \bar{P}_1)} \right)^+, \]
\[ \lim_{\bar{P}_s \to \infty} \frac{R_{2,3}}{\log \bar{P}_s} = m, \]
respectively, where \(m\) is given in (22).
Let us first consider the case where only node 2 helps relay PU’s message, and compute the achievable MG with parameters
\[ \eta_1 = \min \left\{ \frac{C(|h_{14}|^2 \bar{P}_1)}{C(|h_{12}|^2 \bar{P}_1)}, 1 \right\}, \]
\[ \eta_2 = 1 - \eta_1 \quad \text{(i.e., } \eta_2 = 0), \]
\[ \theta_1 = \theta_{14} - \theta_{24}. \]
Notice that, when \(|h_{14}| < |h_{12}|\), node 2 will be able to decode PU’s message with the above choice of \(\eta_1\). In this case, we have \(\eta_2 = 1 - \eta_1 > 0\) and thus, as \(\bar{P}_s \to \infty\), the first constraint in \(S_1\) (c.f. Table I) is satisfied as long as \(h_{23} \neq 0\) and \(\alpha_1 > 0\). This implies that node 3 will also be able to decode PU’s message. In this case, the coexistence constraint (17) becomes
\[ C(|h_{14}|^2 \bar{P}_1) \leq \eta_1 \cdot C(|h_{14}|^2 \bar{P}_1) + (1 - \eta_1) C \left( \frac{|h_{14}| + |h_{24}| \sqrt{\alpha_1 P_2' \bar{P}_1}}{1 + |h_{24}|^2 (1 - \alpha_1) P_2'} \right)^2 \bar{P}_1 \]
\[ = \frac{|h_{14}|^2 \bar{P}_1}{1 + |h_{14}|^2 P_2'}. \quad (34) \]
with \(P_2' = \bar{P}_3/(1 - \eta_1)\). Rearranging the terms in (34), we can see that
\[ |h_{14}|^2 \bar{P}_1 \leq \frac{\alpha_1}{1 - \alpha_1} \]
as \(\bar{P}_s \to \infty\). Thus, the coexistence constraint is satisfied as long as
\[ \alpha_1 \geq \frac{|h_{14}|^2 \bar{P}_1}{1 + |h_{14}|^2 P_2'}. \]
By (12) and (18), we can see that the multiplexing gain achievable in this case is equal to
\[ \eta_2 = 1 - \frac{C(|h_{14}|^2 \bar{P}_1)}{C(|h_{12}|^2 \bar{P}_1)}. \]
Finally, when \(|h_{14}| \geq |h_{12}|\), node 2 will not be able to decode PU’s message and, in this case, node 2 will not be able to transmit its own message without violating the coexistence constraint. The multiplexing gain is zero in this case and, thus, the reason for the non-negative operator (·)+ in (22).
Next, let us consider the case where only node 3 helps relay PU’s message, and compute the achievable MG with parameters \(\eta_1 = \alpha_1 = 0, P_2' = 0, \eta_3 = 1 - \eta_2, \) and \(P_3 = P_2' = \text{...} \)
Notice that Phase 1 is not used in this case and, thus, node 2 will not be able to decode. This fits the condition in Case 3 of Theorem 2, where the achievable rate is given by (20) subject to the constraints in $S_3$. Given the above parameters, the constraints in $S_3$ can be written as

$$\eta_2 \leq \frac{C(h_{14}^2 P_1)}{C(h_{13}^2 P_1)}$$

and

$$C(h_{14}^2 P_1) \leq \eta_2 C\left(\frac{h_{14}^2 P_1}{1 + h_{24}^2 P_s}\right) + (1 - \eta_2) C\left((h_{14}^2 \sqrt{P_1} + h_{34}^2 \sqrt{P_s})^2\right),$$

which can be satisfied only when $|h_{13}| \geq |h_{14}|$. In this case, as $P_s \to \infty$, the constraints on $\eta_2$ can be reduced to

$$\frac{C(h_{14}^2 P_1)}{C(h_{13}^2 P_1)} \leq \eta_2 < 1.$$ 

By (20), we can see that the multiplexing gain achieved in this case is $m = a$, for any

$$a \in \left[\frac{C(h_{14}^2 P_1)}{C(h_{13}^2 P_1)} + 1\right],$$

if $|h_{13}| \geq |h_{14}|$, and is $m = 0$, otherwise.

D. Proof of Theorem 3

Following the steps for proving (17) in Theorem 2 and by applying Lemma 1, we can show that PU’s single-user achievable rate $R_T$ is upper bounded by

$$R_T \leq \frac{1}{n} \sum_{t=1}^{\lfloor \frac{n}{n} \rfloor} \log (1 + |h_{14}(t)|^2 P_1)$$

$$+ \frac{1}{n} \sum_{t=\lfloor \frac{n}{n} \rfloor + 1}^{\lfloor \frac{n}{n} \rfloor + |\eta_{14}| + 1} \log \left(1 + \frac{|h_{14}(t) + \sqrt{\alpha_1 P_2}/\sqrt{h_{24}(t)^2 (1 - \alpha_1)}}{1 + |h_{24}(t)|^2 (1 - \alpha_1) P_2}\right)$$

$$+ \frac{1}{n} \sum_{t=\lfloor \frac{n}{n} \rfloor + 1}^{\lfloor \frac{n}{n} \rfloor + |\eta_{14}| + 1} \log \left(1 + |h_{14}(t) + \sqrt{P_2'/P_1 h_{24}(t) + \sqrt{P_3/P_1 h_{34}(t)^2 P_1}}|\right),$$

where $h_{ij}(t)$ is the realization of the random channel $H_{ij}$ at symbol $t$. When $n$ is large enough, the first term of the RHS of (35) can be rewritten as

$$\frac{1}{n} \sum_{t=1}^{\lfloor \frac{n}{n} \rfloor} \log (1 + |h_{14}(t)|^2 P_1)$$

$$= \eta_1 \frac{1}{n} \sum_{t=1}^{\lfloor \frac{n}{n} \rfloor} \log (1 + |h_{14}(t)|^2 P_1)$$

$$= \eta_1 E[\log (1 + |H_{14}|^2 P_1)],$$

where the last equality comes from the assumption that the channel coefficients are i.i.d. and applying the ergodicity property. After applying the same steps to the other two terms on the RHS of (35), we obtain the upper bound of the single-user achievable rate in (23). Similarly, SU’s achievable rates given in (24)-(26) can be obtained by exploiting Lemma 2. As for the steps to obtain (35), we again invoke Lemma 2 but modify the proof steps of Theorem 2 with the channel coefficients replaced by $h_{ij}(t)$. Then we invoke the channel ergodicity as in (36) to reach (24)-(26). The details are omitted.

E. Proof of Proposition 1

We prove the result by showing that the rate achievable by the linear assignment Gel’fand Pinsker coding (LA-GPC) [26] [27], which is an upper bound of the rate of DPC [10], is the same as the rate achieved when treating interference as noise. It has been shown in [10] that, by treating $X_1$ as the side-information, the rate achievable by LA-GPC is given by

$$\frac{f(\beta)}{\beta} \leq \max \left\{E\left[\log (\bar{P}_1 \bar{P}_2 | h_{13} - \beta h_{23} |^2 + \bar{P}_2 + |\beta|^2 \bar{P}_1)\right] - f(\beta)\right\},$$

where

$$f(\beta) = \frac{E\left[\log (\bar{P}_1 \bar{P}_2 | h_{13} - \beta h_{23} |^2 + \bar{P}_2)\right]}{\bar{P}_1},$$

and $\beta \in C$ is the precoding coefficient of LA-GPC. Note that solving (37) over $\beta$ is the same as minimizing $f(\beta)$. However, since

$$f(0) = E\left[\log (\bar{P}_1 \bar{P}_2 | h_{13} |^2 + \bar{P}_2)\right] \leq E\left[\log (\bar{P}_1 \bar{P}_2 | h_{13} |^2 + \bar{P}_2)\right]$$

$$\leq E\left[\log (\bar{P}_1 \bar{P}_2 | h_{13} - \beta h_{23} |^2 + \bar{P}_2)\right]$$

$$= f(\beta),$$

the function $f(\beta)$ is minimized by choosing $\beta = 0$. In the above, (a) comes from the fact that both

$$1 + |\beta|^2 \sigma^2 e^{2} \sigma^2 e^{13} |h_{13}|^2$$

and

$$|h_{13} - \beta h_{23}|^2$$

have the same distribution since $h_{23}$ and $h_{13}$ are independent zero-mean Gaussian distributed with variance $\sigma^2 e^{2}$ and $\sigma^2 e^{13}$, respectively. Therefore, $\beta = 0$ minimizes $f(\beta)$ and thus maximizes (37). By substituting $\beta = 0$ into (37), the rate given in Proposition 1 is obtained.

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