Problem 1 (20%)  
Consider the signal \( x(t) = A \cos(2\pi f_d t) \).
(a) (5%) Find the average power.
(b) (5%) Find the Fourier transform of \( x(t) \).
(c) (5%) Calculate \( f_s \frac{\partial}{\partial t} \int_0^t x(t) x(t+\tau) d\tau \).
(d) (5%) Find the power spectral density of \( x(t) \).

Problem 2 (15%)  
An FM modulator has output \( x(t) = 100 \cos \left[ 1000\pi t + 2\pi f_d \int_0^t m(\tau) d\tau \right] \) where \( f_d = 25 \text{ Hz/\text{V}} \). Assume that \( m(t) = 4 \cos(40\pi t) \).
(a) (5%) Find the peak frequency deviation.
(b) (5%) Find the modulation index.
(c) (5%) Find the bandwidth of the modulator output by Carson's rule.

Problem 3 (15%)  
A signal \( x(t) \) is multiplied by a periodic pulse train \( c(t) \) described over one period \( T \) as \( c(t) = \begin{cases} 1, & \text{for } |t| \leq S \text{ sec} \\ 0, & \text{for } S \text{ sec} < |t| < T/2 \text{ sec} \end{cases} \).
(a) (10%) What constraint should be placed on \( X(f) \) (the Fourier transform of \( x(t) \)) to ensure that \( x(t) \) can be recovered from the product \( x(t)c(t) \) by using an ideal low pass filter? Find the passband gain of the ideal low pass filter needed to recover \( x(t) \) from \( x(t)c(t) \).
(b) (5%) Consider a set of 20 band-limited signals and each of them has Fourier transform equal to zero for \( |f| \geq 1000 \text{ Hz} \). All 20 signals are to be time-division multiplexed after each is multiplied by a carrier \( c(t) \). If the period \( T \) of \( c(t) \) is chosen to have the maximum allowable value, find the largest value of \( S \) such that all 20 signals can be time-division multiplexed.
Problem 4 (16%)  
Assume that the symbol pertaining to the message points shown in Figure 1 are equally likely.
(a) (8%) Do the two signal constellations shown in Figure 1 (a) and (b) have the same average probability of symbol error? Justify your answer.
(b) (8%) Which of these two constellations has minimum average energy? Justify your answer.

Figure 1

Problem 5 (24%)  
Define \( R_s = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} a_i a_{i+1} \), where \( a_i \) denotes the strength of the \( i \)th pulse. Assume that the strength of a pulse \( p(t) \) is 1, and both 1 and 0 are transmitted equally likely. Find \( R_s \) and \( R_i \) for the following signaling.
(a) (8%) Polar signaling (bit 1 is transmitted by a pulse \( p(t) \) and bit 0 is transmitted by a pulse \(-p(t)\)).
(b) (8%) On-Off signaling (bit 1 is transmitted by a pulse \( p(t) \) and bit 0 is transmitted by no pulse).
(c) (8%) Bipolar signaling (bit 0 is transmitted by no pulse and bit 1 is transmitted by a pulse \( p(t) \) or \(-p(t)\), depending on whether the previous bit 1 was transmitted by \(-p(t)\) or \( p(t)\)).

Problem 6 (10%)  
In a narrow band digital communication system, let \( P_h \) and \( P_q \) denote the probability of symbol error for the in-phase and quadrature channels, respectively. Derive the average probability of symbol error for the overall system.