1. Show that $7x^2$ is $O(x^3)$ and $x^3$ is not $O(7x^2)$. (10%)

2. What are equivalence relations and what are partial ordering relations? (8%) Give an example for each and explain. (8%)

3. The chromatic number of a graph is the least number of colors needed for a coloring of the graph where adjacent vertices have different color. Draw two 6 vertices graph with chromatic number 2 and 3 respectively. (6%)

4. Show that the language $L = \{a^n b^m \mid n \geq m \geq 1\}$ is not a finite state language. (10%)

5. In how many ways can the 26 letters of the alphabet can be permuted so that none of the patterns car, dog, pun, or bye occurs? (8%)

6. Let $G$ be a cyclic group. Show that if $|G|$ is infinite, then $G$ is isomorphic to $(\mathbb{Z}, +)$, where $(\mathbb{Z}, +)$ is a group and $\mathbb{Z}$ is a set of integers. (20%)

7. Let $H_j$ be a harmonic number and $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{j}$ for any positive integer $j$. Show that $\sum_{j=1}^{n} H_j = (n + 1)H_n - n$. (15%)

8. Let $f(1) = 1$ and $f(n) = 2f(\frac{n}{2}) + n - 1$, where $n = 2^k$ and $k$ is a positive integer. Please derive the closed form of $f(n)$. (15%)