1. When a person is accused of a crime, he or she faces a trial. The jury must decide the verdict in the same way that a statistician draws a conclusion about a test of hypothesis. The null and alternative hypotheses are:
   \[ H_0: \text{The defendant is innocent.} \]
   \[ H_1: \text{The defendant is guilty.} \]
   (1) Define when Type I error and Type II error occur. (10%)
   (2) If \( H_0 \) is not rejected, can we conclude that the defendant is innocent? Why yes? Why no? (5%)
   (3) In this case, Type I error is usually more serious. How can we decrease the probability of Type I errors? Is there any side effect in Type II error? (5%)

2. A firm that sells and services minicomputers is concerned about the volume of services calls. The required number of service technicians grows in almost exact proportion to the number of service calls. Discussion with the service manager indicates that the key variables in determining the volume of services calls seem to be the number of computers in use, the number of new installations, whether or not a model change has been introduced recently, and the average temperature. (High temperatures lead to more frequent computer troubles, especially in imperfectly air-conditioned offices.)
   (1) Define and list the dependent variable(s) and independent variable(s) in this case. (10%)
   (2) A major consideration in selecting predictor variables is the problem of collinearity. Define what the collinearity means and what the possible results are. (5%)
   (3) Is collinearity a possible problem in this case? How to avoid it? (5%)

3. A 95% confidence interval for the average passengers on a scheduled flight had been calculated as (102.80, 121.20). Does the confidence interval include the true value of the parameter being estimated? What does the 95% mean? (10%)

4. A known probability density function is given as: (14%; 2% for each)
   \[ f(x) = c x (1 - x) \quad 0 < x < 1 \]
   \[ 0 \quad \text{others} \]
   where \( c \) is a constant and \( X \) is a random variable.

Find the following:
   a. \( c \)
   b. Expected value \( \mu \)
   c. Variance \( \sigma^2 \)
   d. Median \( \eta \)
   e. Mode \( M_0 \)
   f. \( P( X \leq 0.2 ) \)
   g. \( P( \mu - 2 \sigma \leq X \leq \mu + 2 \sigma ) \)
5. Find the cumulative distribution function, $F(x)$, for each of the following distributions:
(12%; 2% for each)

a. $f(x) = 1, \quad x = 0$
b. $f(x) = 1/3, \quad x = -1, 0, 1$
c. $f(x) = x/15, \quad x = 1, 2, 3, 4, 5$
d. $f(x) = 3(1 - x)^2, \quad 0 < x < 1$
e. $f(x) = 1/x^2, \quad 1 < x < \infty$
f. $f(x) = 1/3, \quad 0 < x < 1 \text{ or } 2 < x < 4$

6. Let $X$ and $Y$ be two normally independent random variables with expected values and
variances for $E(X) = 5, \quad \sigma^2(X) = 6$ and $E(Y) = -5, \quad \sigma^2(Y) = 10$, respectively. If $W$ is
a random variable and $W = X + Y$, then answer or find:
(8%; 2% for each)

a. What distribution is it for $W$?
b. $E(W)$ and $\sigma^2(W)$
c. Skewness $\beta_1$
d. Kurtosis $\beta_2$

7. The birth phenomenon of the married women (over the age of 45) for a specific
district is given as:

| Children number | 0   | 1   | 2   | 3   | 4   | 5   | 6
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</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.16</td>
<td>0.17</td>
<td>0.24</td>
<td>0.16</td>
<td>0.10</td>
<td>0.00</td>
<td>0.17</td>
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Answer or find the following: (16%; 4% for each)

a. The average children number for each woman? (use mean $\mu$, median $\eta$ and
mode $M_0$ to explain your answer)
b. Coefficient of variation?
c. The most common children number for each family?
d. The probability of the family with at least 2 children?