1. (50%) Consider a PID control system shown in Figure P1,

\[ G(s) = \frac{k}{(1+\tau_i s)(1+\tau_1 s)} \]

\[ D(s) = 1 + \frac{k_d}{s} \]

\[ F(s) = (1 + k_c s) \]

\[ \tau_i = 1, \quad \tau_1 = 0.1, \quad k_d = 0.01, \quad k_c = 0.1. \]

(a) (10%) Find the closed-loop transfer function?

(b) (10%) Find the steady-state errors for unit step input and unit ramp input?

(c) (10%) Sketch the Root Locus for \( k > 0 \)

(d) (10%) Sketch the Nyquist Plot for \( k = 10 \).

(e) (10%) Plot the open-loop Bode Diagram, and find the gain margin and phase margin for \( k = 10 \).
2. Compute the inverse Laplace transform of the following: (10%)

\[
\frac{s+1}{(s-1)(s+2)}
\]

3. (a) If we apply \( u(t) = \sin \pi t \) to a device, called a half-wave rectifier, then the output consists of the part of \( u(t) \) with \( u(t) > 0 \), that is

\[
y(t) = \begin{cases} 
\sin \pi t & \text{for } 2k \leq t < 2k + 1 \\
0 & \text{for } 2k + 1 \leq t < 2(k + 1)
\end{cases}
\]

for \( k = 0, \pm 1, \pm 2, \ldots \). Find the Fourier series of \( y(t) \). (10%)

(b) What are the average powers of \( u(t) \) and \( y(t) \)? (5%)

(c) What percentage of the input power is transmitted to the output? (5%)

4. Find a set of \( \alpha_i \), \( i = 1, 2, 3, 4 \), so that the function

\[
\overline{f(t)} = \alpha_1 \phi_1(t) + \alpha_2 \phi_2(t) + \alpha_3 \phi_3(t) + \alpha_4 \phi_4(t)
\]

where \( \phi_i(t) \) are as defined in Figure P4, minimizes

\[
\int_0^T [\sin t - \overline{f(t)}]^2 dt
\]

(20%)

Figure P4