共四題，每題25分，可不依序作答，但題號請務必標示清楚。解題時請註明所依據之定律或原理；自由體圖須簡明繪於答覆卷上。若需自行定義代號或向量，亦請於圖上標示清楚。重力加速度 \((g)\) 之值請以 9.81 m/s² 或 32.2 ft/s² 計算。

1. A solid sphere A with a radius of \(R\) has an initial linear velocity \(v_0\) and an initial angular velocity \(\omega_0\), as shown in Fig. 1. Another solid sphere B of identical radius and mass is sitting at rest at a distance of \(S\) away from sphere A. Now, sphere A is traveling on a rough horizontal surface toward sphere B and is going to make direct central impact with sphere B. After the impact, sphere A is standing still at the spot. Assume that the friction between spheres is negligible and the impact is perfectly elastic. Denoting by \(\mu_k\) the coefficient of kinetic friction between the sphere and the floor, please answer the following questions:

(a). Determine the linear velocities, \((v_B)_f\), and angular velocity, \((\omega_B)_f\), of sphere B immediately after the impact? Please express them in terms of \(v_0\) and \(\omega_0\).

(b). If \(\omega_0\) is unknown and \(S\) is given, with other conditions unchanged, find \(\omega_0\) as function of \(S\).

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![Fig. 1](image-url)
2. A 0.6-kg mass slides on a smooth horizontal surface at the end of an inextensible string (Fig. 2). The other end of the string passes through a smooth hole in the surface and is attached to a spring having $k = 100 \text{ N/m}$. The spring is unstretched when $\ell = 0$. If $v = 10 \text{ m/s}$ and $\ell = 0.5 \text{ m}$ at the instant shown,

(a). determine the minimum and maximum values of $\ell$ in the resulting motion.

(b). explain the physical phenomena when the string is at its maximum and minimum length.
3. The slotted disk sector rotates with a constant counterclockwise angular velocity \( \omega = 4 \text{ rad/s} \). Simultaneously the slotted arm OC oscillates about the line OB (fixed to the disk) so that \( \theta \) changes at the constant rate of \( 2 \text{ rad/s} \) except at the extremities of the oscillation during reversal of direction. Determine the magnitude of the total acceleration of the pin A when \( \theta = 30^\circ \) and \( \theta \) is positive (clockwise). (Fig. 3)

\[
\frac{d}{dt} \cos \theta = -\dot{\theta} \sin \theta ; \quad \frac{d}{dt} \sin \theta = \dot{\theta} \cos \theta ; \quad \frac{d}{dt} \tan \theta = \dot{\theta} \sec^2 \theta ; \quad \frac{d}{dt} \sec \theta = \dot{\theta} \sec \theta \tan \theta
\]

![Fig. 3](image-url)
4. The guide with the vertical slot is given a horizontal oscillatory motion according to
\[ x = 100 \sin 2t \] where \( x \) is in millimeters and \( t \) is in seconds. The oscillation causes the
pin \( P \) to move in the fixed slot whose shape is given by
\[ y = \frac{x^2}{100} + \frac{0.2x}{x + 10} \text{ with } y \]
also in millimeters. Find and locate the maximum value of the velocity of the pin
between 0 and 1 second. Please solve the problem numerically by using a time step of
0.001 second. Write your computer program in one of the following languages:
Fortran, Basic, or C. (Fig. 4)