1. Solve

\[
2 \frac{dx}{dt} + \frac{dy}{dt} - y = t
\]
\[
\frac{dx}{dt} + \frac{dy}{dt} = t^2
\]

subject to \( x(0) = 1, \ y(0) = 0. \) \hspace{1cm} (15%) 

2. Solve

\[
\frac{dx}{dt} = -4x + y + z
\]
\[
\frac{dy}{dt} = x + 5y - z
\]
\[
\frac{dz}{dt} = y - 3z
\] \hspace{1cm} (15%) 

3. Use polar coordinates to evaluate

\[
\int_0^1 \int_0^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2} \, dy \, dx
\] \hspace{1cm} (10%) 

4. Find the Fourier series of the following function which is assumed to have the period \( 2\pi \).

\[ f(x) = \frac{x^2}{4} \quad (-\pi < x < \pi) \] \hspace{1cm} (10%) 

5. Prove that if \( f(t) \), defined for all positive \( t \), is a periodic function with period \( T \), that is, \( f(t+nT) = f(t) \) for all integers \( n \), then

\[ \mathcal{L}\{f(t)\} = \frac{1}{1-e^{-\pi}} \int_0^{\pi} e^{-\pi f(t)} \, dt \] \hspace{1cm} (10%)
6. Find the solution \( u(x,t) \) of the problem

\[
\begin{align*}
    u_{tt} &= c^2 u_{xx} + h, & 0 < x < L, \ t > 0 \\
    u(x,0) &= 0 \\
    u_t(x,0) &= 0 \\
    u(0,t) &= 0 \\
    u(L,t) &= 0 \\
\end{align*}
\]

where \( c \) and \( h \) are constants. (20%) 

7. Assuming that the mechanical system shown below is at rest before excitation force \( P \sin \omega t \) is given. Using the Laplace transform method, derive the complete solution \( x(t) \). The displacement \( x \) is measured from the equilibrium position before the excitation force is given. (20%)