1. (a) Write the negation of
    "For every x, if x ∈ A, then x ∈ B."

(b) Write the contrapositive of
    "(x,y) ∈ R and (y,x) ∈ R imply that x = y."

(c) Prove or disprove:
    "If (q ∧ r) → p and q → ¬ r, then p."

(10%)  

2. Let $R_1$ be a relation on \{1,2,3,4\} whose Boolean matrix is

\[
R_1 = \begin{pmatrix}
1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 \\
\end{pmatrix}
\]

(a) Determine whether $R_1$ is an equivalence relation, or a partial order, or neither. Explain your answer.

(b) If $R_1$ is an equivalence relation, then find the corresponding equivalence classes.

   If $R_1$ is a partial order, then extend $R_1$ to a total order that preserves the original ordering.

   If $R_1$ is neither an equivalence relation nor a partial order, then find the smallest equivalence relation containing $R_1$.

(10%)  

3. Let $R_2$ be a relation on \{a,b,c,d,e\} whose Boolean matrix is

\[
R_2 = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

(a) Determine whether $R_2$ is an equivalence relation, or a partial order, or neither. Explain your answer.

(b) If $R_2$ is an equivalence relation, then find the corresponding equivalence classes.

   If $R_2$ is a partial order, then extend $R_2$ to a total order that preserves the original ordering.

   If $R_2$ is neither an equivalence relation nor a partial order, then find the smallest equivalence relation containing $R_2$.

(10%)
4. Given group \( (G, *) \).
   The subgroup of \( (G, *) \) generated by \( A \), denoted by \( <A> \), is defined as:
   
   (B) \( A \subseteq <A> \).
   (R1) If \( g, h \in <A> \), then \( g * h \in <A> \).
   (R2) If \( g \in <A> \), then \( g^{-1} \in <A> \).

   (a) Consider the group \( (\mathbb{Z}(12), +_{12}) \).
   Find \( <\{3\}> \) in \( (\mathbb{Z}(12), +_{12}) \). (i.e., find the subgroup of \( (\mathbb{Z}(12), +_{12}) \)
   generated by \( \{3\} \)).

   (b) Determine whether \( <\{3\}> \) is isomorphic to \( (\mathbb{Z}(4), +_4) \) or not. Why?

   (c) Let \( H \) be a subgroup of a group \( (G, *) \). A left coset of \( H \) in \( G \) is a subset of the
   form \( g * H = \{ g * h : h \in H \} \).
   Find the left coset \( 1 * +_{12} <\{3\}> \) of \( <\{3\}> \) in \( (\mathbb{Z}(12), +_{12}) \).

   (d) Show that the left cosets of \( <\{3\}> \) in \( (\mathbb{Z}(12), +_{12}) \) form a partition of
   \( \mathbb{Z}(12) \). (20%)
5. For each of the following six program fragments, give an analysis of the running time in Big-Oh notation:

(1) \( \text{Sum} = 0; \)
\[ \text{for}( i = 0; i < N; i++) \]
\[ \text{Sum}++; \]

(2) \( \text{Sum} = 0; \)
\[ \text{for}( i = 0; i < N; i++) \]
\[ \text{for}( j = 0; j < N; j++) \]
\[ \text{Sum}++; \]

(3) \( \text{Sum} = 0; \)
\[ \text{for}( i = 0; i < N; i++) \]
\[ \text{for}( j = 0; j < N \times N; j++) \]
\[ \text{Sum}++; \]

(4) \( \text{Sum} = 0; \)
\[ \text{for}( i = 0; i < N; i++) \]
\[ \text{for}( j = 0; j < i; j++) \]
\[ \text{Sum}++; \]

(5) \( \text{Sum} = 0; \)
\[ \text{for}( i = 0; i < N; i++) \]
\[ \text{for}( j = 0; j < i \times i; j++) \]
\[ \text{for}( k = 0; k < j; k++) \]
\[ \text{Sum}++; \]

(6) \( \text{Sum} = 0; \)
\[ \text{for}( i = 1; i < N; i++) \]
\[ \text{for}( j = 1; j < i \times i; j++) \]
\[ \text{if}( j \times j = 0) \]
\[ \text{for}( k = 0; k < j; k++) \]
\[ \text{Sum}++; \]

6. A majority element in an array, \( A \), of size \( N \), is an element that appears more than \( (N/2) \) times. Here is a sketch of an algorithm to solve the problem:

Step 1: find a candidate majority element.
To find a candidate in the array, \( A \), form a second array, \( B \). Then compare \( A[1] \) and \( A[2] \). If they are equal, add one of these to \( B \); otherwise do nothing. Then compare \( A[3] \) and \( A[4] \). Again if they are equal, add one of these to \( B \); otherwise, do nothing. Continue in this fashion until the entire array \( A \) is read. Then recursively find a candidate for \( B \); etc.

Step 2: determine if the candidate found is actually the majority.
Do a sequential search through the array \( A \) to verify.
(a) How do you handle the case when there is no majority element at all?
(b) What is the running time of the algorithm?
7. A binary search tree supports search, minimum, successor, etc, operations.
(a) Give the pseudo-code for the minimum(x) operation, given x as a pointer to a node of the
tree.
(b) Give the pseudo-code for the successor(x) operation, given x as a pointer to a node of the
tree, using minimum(x) as a subroutine.
(c) Argue that if a node in a binary search tree has two children, then its successor has no left
child and its predecessor has no right child.

15% 

8. Given input \{371, 323, 173, 199, 344, 679, 989\} to be hashed into a table of size 10, and a
hash function \( h(x) = x \mod 10 \). Show the resulting hash table contents when each of the
following hashing strategies are used.
(a) open addressing with linear probing,
(b) open addressing with quadratic probing, \( h'(x, i) = h(x) + i + i^2 \),
(c) open addressing with second hash function \( h'(x) = (x \mod 9) + 1 \).

10%