1) Let \( D = \text{depth} \)
   a) A general tree contains 235 nodes. What can you determine about the depth of the tree.
      (5%)
   b) A general tree of degree 4 contains 235 nodes. What can you say about the depth of
      the tree. (5%)
   c) A binary tree contains 235 nodes. What can you determine about the depth of the tree.
      (5%)
   d) A binary tree contains 235 nodes and 57 of them have degree 0. What can you say
      about the number of nodes of degree 1. (5%)

2) Determine the frequency counts for all statements AND time complexities for the
   following two programs:

   a) (10%)
      
      \[
      \text{for } i \leftarrow 1 \text{ to } n \\
      \quad \text{for } j \leftarrow 1 \text{ to } i \\
      \quad \quad \text{for } k \leftarrow 1 \text{ to } j \\
      \quad \quad \quad x \leftarrow x + 1 \\
      \quad \quad \text{end} \\
      \quad \text{end} \\
      \text{end} \\
      \]

   b) (5%)
      
      \[
      i \leftarrow 1 \\
      \text{while } i \leq n \text{ do} \\
      \quad x \leftarrow x + 1 \\
      \quad i \leftarrow i + 1 \\
      \text{end} \\
      \]

3) QUICKSORT, HEAPSORT, and the 2-WAY MERGE SORT all have the same average
   time complexity of \( O(n \log n) \). What arguments can you give (e.g., application conditions)
   for each of them that would lead you to make decision on the sorting method to use. (15%)
4) Suppose that the binary tree in Fig. 1 is a splay tree.
   a) Draw the tree obtained after the key 5 is referenced and splayed. (5%)
   b) Draw the tree obtained after the key 8 is inserted into the tree in Fig. 1 and the
      splaying operation is done. (5%)

```
    7
   /|
  4 10
 /|
3 6 9
|
5
```
Fig. 1

5) Suppose that the binary tree in Fig. 1 is an ordinary search tree.
   a) Draw two possible result trees if the root node in Fig. 1 is deleted. Note that the trees
      you draw must be obtainable from a general deletion procedure. (5%)
   b) List the keys in Fig. 1 with level-order and post-order tree traversal, respectively. (5%)
   c) If recursive procedures are not allowed, what data structures should be used to
      implement level-order and post-order traversing, respectively? (5%)

6) The sequence of keys, 19, 31, 41, 9, 30, 21, 62, 17, is to be hashed into a hash table of size
   11 by using the hashing function, $H(x) = x \mod 11$.
   a) Show the content of the hash table obtained if quadratic probing is adopted to resolve
      collisions. (5%)
   b) Show the content of the hash table obtained if double hashing is adopted and the
      increment is determined by the function, $Inc(x) = 9 - x \mod 9$. (5%)
   c) Suppose that random probing is adopted to resolve collisions and $\lambda$ is the loading
      factor. Estimate the expected number of probing needed to insert a key into the hash
      table. (5%)

7) Write a recursive procedure, $int Height(tree T)$, to determine and return the height of the
   binary tree pointed by $T$. Note that a tree node has three fields which are named as LSP
   (pointing to its left son), RSP (pointing to its right son), and KEY. (10%)