1. Two pendulums are suspended from frictionless pivots and connected at their mid-points by a spring. Assume that each pendulum can be represented by a mass M at the end of a massless bar of length L. Also assume that the displacement is small and linear approximations can be used for sinθ and cosθ. The spring located in the middle of the bars is unstretched when θ₁ = θ₂. The input force is represented by F(t), which influences the left-hand bar only. The moment of inertia for a pendulum about the pivot is given by J = ML². Determine the transfer function \( T(s) = \frac{θ_1(s)}{F(s)} \). (15%)

2. Consider the following feedforward system. Determine \( K_f \) such that the steady-state error is zero with a ramp input of \( 1.0t \). (15%)

3. The portion of the system enclosed by the dashed box in the following figure represents the plant to be controlled. For \( K_1 = 2 \), construct the root-locus plot for the entire system, \( 0 < K_2 < \infty \). For what values of \( K_2 \) does the system have an oscillatory response? What is the smallest possible damping ratio? (20%)
4 Shown in Figure is the control system for one joint of a robot arm. The controller is a PD compensator, with $G_C(s)=K_P+K_Ds$. $K_P$ is the proportional gain and $K_D$ is the derivative gain.

(a) Find the transfer function $\frac{\theta_C(s)}{E_d(s)}$. (15%)

(b) As a first step in the system design, determine all possible values for the coefficients $K_P$ and $K_D$, which add a zero to the system at $s = -2$ and simultaneously stabilize the system. (10%)

5 A sketch of a laboratory experiment in a fluid (water) flow is shown in the figure below with the inflow rate $Q_{in}$.

Assume that the flow through the equal-sized holes A, B and C is described by $Q = K \sqrt{P_{in} - P_{out}}$, where $Q$ is the flow rate, $K$ is the constant, $P$ is the pressure.

With holes at A and C and no hole at B, write the equations of motion for this system in terms of $h_1$ and $h_2$. Assume that $h_3$ is 20 cm and $h_2$ is less than 20 cm. When $h_2$ is 10 cm, the outflow is 200 cm$^3$/min. Compute the constant $K$ and include it in your equations. (15%)

6 Can a right-half s-plane pole of a plant be effectively cancelled by a compensator with a right-half s-plane zero? Why? (10%)