1. Four people, a, b, c, and d, were in a race. Right before the race started, Dave said that he thought a would be the second and d would be the fourth, John thought a would be the first and b would be the second, and Paul thought c would be the second and d would be the fourth. If each of them had only one estimate right
   a. Write all the estimates in logical propositions (Use the propositions: A = "a is the first", B = "b is the second", C = "c is the third", D = "d is the fourth", P = "c is the second", Q = "a is the second").
      (5%)
   b. Use the logical operators (formal proof) to find the rank of a, b, c, and d.
      (10%)  

2. Find a recurrence relation for the number of ways to fully parenthesize the expression
   \[ w_1 + w_2 + w_3 + \ldots + w_n \]
   so that only two terms are added at a time; that is, parentheses are used to indicate the order in which the additions are performed. For example, the expression \([(w_1 + w_2) + w_3]\) is fully parenthesized, but \[(w_1 + w_2) + w_3\] is not.
   (10%)  

3. Let \( Z(4) = \{0, 1, 2, 3\} \). Define the binary operator \( \oplus \) on \( Z(4) \) by
   \[ x \oplus y = xy \pmod{4} \]
   for \( x, y \) in \( Z(4) \).
   a. Give the complete table of operator \( \oplus \) for \( Z(4) \).
      (5%)
   b. Determine and justify whether \( (Z(4), \oplus) \) is a semigroup, a monoid, or a group.
      (5%)  

4. Let \( G \) be a planar Hamiltonian simple graph with \( n \) vertices, and let \( C \) be a Hamiltonian cycle in \( G \). Then with respect to \( C \), prove \( \Sigma (k - 2)(t_k - s_k) = 0 \). Here \( t_k \) is the number of faces inside \( C \) whose boundary contain exactly \( k \) edges, and \( s_k \) is the number of faces outside \( C \) whose boundary contains exactly \( k \) edges.
   (15%)  

5. Let \( M \) be the adjacency matrix of the simple graph \( G \),
   a. Show that the number of triangles in \( G \) is \( \frac{1}{6} \| M^3 \| \). Here \( \| M^3 \| \) denotes the sum of the elements of the main diagonal of \( M^3 \).
      (5%)
   b. Use the above formula, find the number of triangles in the following graph:
      (5%)  

6. Write the trace of the following program using the following tree.
   Program:
   ```c
   void xx ( pointer ptr )
   { if (ptr) { xx (ptr -> left );
       xx (ptr -> right );
       printf("%d", ptr -> data );
   }
   }
   ```
   Tree:
   ```c
   +
   /|
   / |
   /  |
   -E-
   / |
   /  |
   /  |
   -D-
   / |
   /  |
   /  |
   -C-
   / |
   /  |
   /  |
   -A-
   ```
   (10%)
7. The following is a quicksort program.

```c
void quicksort ( int i, int j )
    /* sorts the records stored in array data between indexed by i and j in descending order. */
    { int p;
        if (i < j)
            { p = partition ( i, j );
                quicksort ( i, p-1 );
                quicksort ( p+1, j );
            }
    }

Write the algorithm of the inside function partition( i, j ). (15%)
```

8. Describe the functions(and the parameters) of the following three parts:
   i. void add ( pointer *ptr, pointer node)
      { ...
        node -> link = *ptr;
        *ptr = node;
      }
   (5%)

   ii. void add (pointer *head, pointer *ptr, pointer node)
      { ...
        node -> link = NULL;
        if ( *head ) (ptr) ->link = node;
        else *head = node;
        *ptr = node;
      }
   (5%)

   iii. void add ( pointer *ptr, pointer node)
      { ...
        if (IS_EMPTY (*ptr))
            { *ptr = node;
              node ->link = node;
            }
        else { node-> link = ( *ptr ) -> link;
               ( *ptr ) -> link = node;
            }
      }
   (5%)