1. a) $f(t) = e^{-at}$, find the Fourier transform of $f(t)$ given that $\int_{-\infty}^{\infty} e^{-at} \, dt = \frac{1}{a}$ (10%)

b) Evaluate $\int_{-\infty}^{\infty} \delta(t) \sin(t) \, dt$ (5%)

c) Is that $\delta(t)$ an odd or even function? Prove it (5%)

2. a) Show that if $M + Ny = 0$ has a solution, then it has an integrating factor (5%)
b) Solve $(x + y) \, dx + x \, \ln x \, dy = 0$ (10%).

3. The Laplace transform of $x(t)$ is $X(s)$ prove that $\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$ (10%)

4. Convolution theorem states that the relationship among $h(t)$, $x(t)$, and $y(t)$ is $y(t) = x(t) * h(t)$, where $h(t)$, $x(t)$, and * denote the system impulse response, input, output and convolution operation, respectively. If $x(t) = 1 - \cos(\pi t)$, $h(t) = e^{\pi t}$, find $y(t)$ (10%).

5. If $A = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 2 & 6 \\ 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ find the eigenvalues of $A$ and $B$.

(Hint: decomposing the matrices into small size submatrices) (10%)

6. $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$ changing $A$ to an orthogonal matrix (10%)

7. If $\vec{v} = x\hat{i} + (xy)\hat{j}$
a) Does it exist $\phi(x, y)$ such that $\nabla \phi(x, y) = \vec{v}$? If yes, find $\phi(x, y)$ (5%).
b) Find $\int_{c} \vec{v} \cdot d\vec{r}$, where $c$ is $x + y^2 = b^2$. Is this independent of path? (5%)

8. Evaluate $\int_{\pi}^{\pi} \frac{\cos 2\theta d\theta}{1 - 2p\cos \theta + p^2}$ where $-1 < p < 1$. (15%)