注意事項：
1. 本試題共 [4] 題，滿分共 100 分。請按順序標明題號作答，不抄題。
2. Show all your calculations.

(25 %)
1. Let \( X \) and \( Y \) have the joint probability density function
\[
f(x, y) = c \cdot e^{-(x+y)}, \quad 0 \leq x \leq 2y < \infty
\]
(1) Determine the appropriate value of \( c \). (5 %)
(2) What is the marginal distribution of \( Y \)? (6 %)
(3) Compute the value of \( E(X) \). (6 %)
(4) Find the probability density function of \( W = X + Y \). (8 %)

(25 %)
2. Answer the following questions if a sample contains five values 1.2, 2.3, 3.9, 4.6, and 5.0.
(1) Which one of the following uniformly distributions is the possible answer to generate above sample? Explain your answer clearly. (7 %)
   (a) \( f(x) = \frac{1}{\theta}, \quad \theta \leq x \leq 2\theta \)
   (b) \( f(x) = \frac{1}{2\theta}, \quad \theta \leq x \leq 3\theta \)
   (c) \( f(x) = \frac{1}{3\theta}, \quad \theta \leq x \leq 4\theta \)
   (d) \( f(x) = \frac{1}{4\theta}, \quad \theta \leq x \leq 5\theta \)
(2) If above sample is assumed to be from the distribution
\[
f(x) = \frac{2x}{\theta^2}, \quad 0 \leq x \leq \theta
\]
   What are the maximum likelihood estimator (say \( \hat{\theta} \)) and estimate of the parameter \( \theta \) for this sample? (6 %)
(3) Derive the density function for \( \hat{\theta} \). (6 %)
(4) Determine the value of \( c \) such that \( c\hat{\theta} \) is the unbiased estimator of \( \theta \). (6 %)

(30 %)
3. For the simple linear model, show that \( SSE = S_{yy} - \hat{\beta}S_{xy} \) (15%) and \( E[SSE/(n-2)] = \sigma^2 \) (15%)
where \( \hat{\beta} = S_{xy} / \sum_{i=1}^{n} (x_i - \bar{x})^2 \), \( S_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})^2 \) and \( S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \).

(20 %)
4. Suppose that \( X \sim \text{Poisson}(\mu) \). Derive the most powerful test \( H_0 : \mu = \mu_0 \) versus \( H_1 : \mu = \mu_1 (\mu_1 > \mu_0) \) based on an observed value of \( X \).