1. The following plant model is used to describe the speed control system

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
-1 & 1 \\
-10 & -1
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix}
+ \begin{bmatrix}
0 \\
10
\end{bmatrix} u(t)
\]

\[y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\
x_2(t)\end{bmatrix}\]

where

\[x_1(t) = \omega(t), \text{ angular velocity of the motor shaft}\]

\[x_2(t) = i_a(t), \text{ armature current}\]

Evaluate the response of this system, \( y(t) \), to a unit step input \( u(t) = 1 \) for \( t \geq 0 \) under zero initial conditions. (15%) 

2. The input signal to an audio amplifier is given by \( y(t) = 10 \cos \omega_0 t \). However, the amplifier, being nonlinear at higher amplitude levels, clips all amplitudes beyond ±8 as shown in the below figure.

Find the Fourier series of the distortion signal, \( y_d(t) \) (20%)
3. A thin bar of length $\pi$ units is placed in boiling water (temperature 100°C). After reaching 100°C throughout, the bar is removed from the boiling water. With the lateral sides kept insulated, suddenly, at time $t = 0$, the ends are immersed in a medium with constant freezing temperature 0°C. The temperature distribution $u(x,t)$ in the bar at time $t$ satisfies the one dimensional heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$

Find the temperature distribution $u(x,t)$ for $t > 0$.

4. Solve the following differential equations:
   (a) $xy'' = x^2 + y^2, \quad x, y \neq 0$ 
   (b) $y''' - 4y'' - 7y' + 10y = 3x - e^x$

5. (a) State what the Cauchy Integral Formula is.
   (b) Use the Cauchy Integral Formula to find $\int_{\Gamma} \frac{z \sin(2z-3i)}{(z+i)} \, dz$ where $\Gamma$ is the circle $|z-i|=3$.

6. About the power series expansion, answer the following:
   (a) (True or False) A function $f(z)$ is analytic at a point $z_0$ if and only if the function is infinitely differentiable at $z_0$.
   (b) (True or False) If a function $f(z)$ is analytic at a point $z_0$, a power series expansion of such function at that point exists, and it will be the Taylor series expansion.
   (c) (True or False) The Maclaurin series is a special case of the Taylor series where the expansion point is at the origin.
   (d) Explain the "ratio test" and its purpose.
   (e) Use the power series expansion about the initial condition point to solve the following initial value problem, list the first five nonzero terms: $y'' + xy' = -1 + x, \quad y(2) = 1, \quad y'(2) = -4$