1. Find the general solution of 
\[ y'' + 2y' + y = e^{-x} \ln x \] 
(20%) 

2. Solve the integral equation 
\[ f(t) = \cos(t) + e^{-2t} \int_0^t f(\alpha) e^{2\alpha} d\alpha \] 
(20%) 

3. (i) Find the matrix inverse \( A^{-1} \) of matrix 
\[ A = \begin{pmatrix} 1 & -2 & 4 \\ -1 & 1 & -3 \\ 1 & -1 & -4 \end{pmatrix} \] 

(ii) For a system of linear equations \( AX = B \), in which \( A \) is the \( n \times n \) matrix of coefficients, \( X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \) and \( B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \). Discuss the conditions for the system to have (a) no solution; (b) one solution; (c) more than one solution 
(20%) 

4. Evaluate the integral 
\[ \oint_C z \, dx \] 
where \( C \) is the circle \( r = a \) (\( a > 0 \)). 
(20%) 

5. Solve the boundary value problem 
\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{for} \quad 0 < x < 1, \quad t > 0 \] 
\[ u(0,t) = u(1,t) = 0 \quad \text{for} \quad t \geq 0 \] 
\[ u(x,0) = 1 \quad \text{for} \quad 0 \leq x \leq 1 \] 
(20%)